

## A NEW HEAT TRANSFER CORRELATION FOR OSCILLATING FLUID FLOW

by

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*In this study, heat transfer under oscillating flow conditions has been investigated conducting a wide range of experiments using water as the flow medium. Experimental data have been reduced to obtain relevant heat transfer parameters, namely Nusselt number, and compared to those obtained by the previous researchers. The new correlation obtained herein covers a wide range of flow conditions and it takes into account the effect of Prandtl number which is very sensitive to temperature variations. It is believed to be a good representation of the physical problem. A total of 27 sets of heat transfer experiments, each representing a different frequency, flow displacement length and heat input, have been conducted. Heat applied is absorbed by concentric-pipe heat exchanger where calculations have been performed.*

Key words: *oscillating flow, heat transfer*

### Introduction

Oscillating flow is a physical phenomenon encountered in internal combustion engines, Stirling engines, cryogenic coolers and chemical processes. Due to the scarcity of experimental data, this physical phenomenon has not yet been fully understood. Oscillating flows can be either pulsating where flow oscillates around a non-zero average or reciprocating where flow changes its direction at every half cycle. The average velocity in reciprocating flow is zero. Pulsating flow has been studied extensively in Medical and Biological sciences. This type of fluid flow can also be observed at the outlet of a piston pump, suction and exhaust manifolds of internal combustion engines, hydraulic and pneumatic lines, and control systems. On the other hand, reciprocating flows can be observed in many applications such as internal combustion engines, Stirling engines and cryogenic coolers.

The fact that oscillating flow has two thermal entrance regions is one of the reasons why it enhances heat transfer. In the past studies, the effect of oscillation on the heat transfer enhancement was investigated analytically and experimentally. These studies include self oscillation and forced oscillation.

Akdag *et al.* [1] have shown that the heat transfer in a sinusoidally oscillating fluid column can be modelled using the control volume approach.

Ozdemir and Ozguc [2] have shown that the heat added and absorbed in a Fluidyne heat engine must be equal in order for the system to be stable (except friction).

Akdag and Ozdemir [3] have found a correlation for average Nusselt number using the data for an annular vertical fluid column open to atmosphere and shown its strong dependence on kinetic Reynolds number.

Arslan and Ozdemir [4] have shown that the system consisting of an oscillating heat pipe made up of three interconnected fluid columns has two degrees of freedom and the second frequency is double the first one.

Akdag *et al.* [5] have found a correlation regarding heat absorption from a vertical annular fluid column using the control volume approach. It has been shown that the Nusselt number increases with increasing frequency.

Zhao and Cheng [6] have conducted heat transfer experiments for sinusoidal flow in a finite-length pipe and have shown that the heat transfer can be characterized with kinetic Reynolds number, non-dimensional oscillating fluid displacement, length/diameter ratio of pipe and the Prandtl number.

Shahin [7] has shown in his Ph. D. thesis that in an annular oscillating flow between two concentric pipes, Nusselt number increases with increasing frequency up to a certain value.

Walther *et al.* [8] have investigated the heat transfer associated with turbulent oscillating flow conditions as found in heat exchangers of regenerative thermal machines based on numerical flow modelling and a suitable low-Reynolds number  $k-\varepsilon$  turbulence model.

Zhao and Cheng [9] have carried out an experimental study for laminar oscillatory forced convection in a long circular tube heated by uniform heat flux and subjected to a laminar reciprocating flow of air. The numerical solutions for time-resolved centerline fluid temperature, cycle-averaged wall temperature, and the space-cycle averaged Nusselt number are shown to be in good agreement with the experimental data. Based on the experimental data, a correlation equation for the space-cycle averaged Nusselt number of an oscillatory laminar flow of air in a long tube in terms of appropriate similarity parameters is obtained.

Bouvier *et al.* [10] have built an instrumented test rig which allowed them to study heat transfer in oscillating flow. They have shown that fundamental frequencies are not the same in the wall and in the fluid.

### Experimental test rig

A schematic diagram of the experimental test rig is shown in fig. 1. It is made up of a heated section where heat is applied circumferentially at various levels, a double acting piston-cylinder assembly made from stainless steel used as oscillation generator and two concentric heat exchangers installed symmetrically which are also made from stainless steel. Oscillation is obtained using a flywheel-crank assembly that is driven by a motor-reducer. The rotational speed is changed by means of an A/C motor drive that converts the line frequency of 50 Hz to a desired value. Frequencies 5, 10, and 15 Hz have been used in this work that correspond to piston frequencies of 0.116 Hz, 2.330 Hz, and 3.495 Hz, respectively. Two constant temperature baths (CBT) with circulation pumps, each having a maximum cooling capacity of 300 W, are connected to the heat exchangers provide cooling water and share the heat load equally.

Temperature data have been collected using a Keithley 2700 data acquisition system and a computer analyzes the collected data. All thermocouples are K-type (Ni-Cr/Ni-Al). Temperature data collected have been time averaged over the number of cycles each set of experiment was conducted. Thus, all the calculations have been performed using these time averaged temperature values.

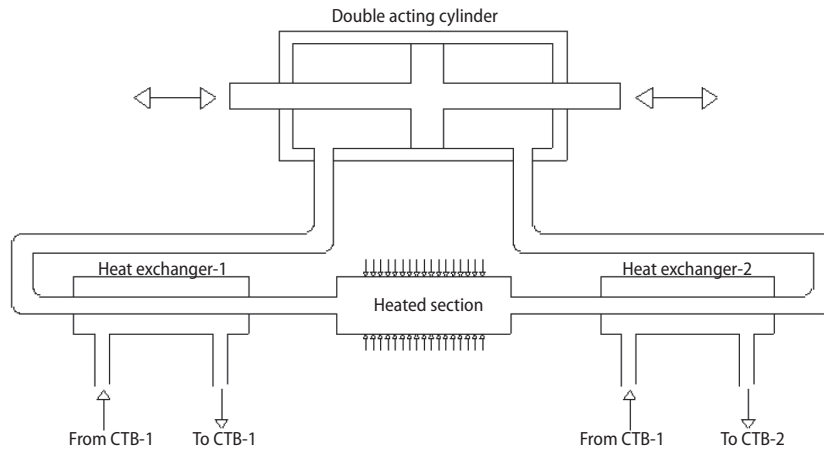


Figure 1. Experimental test rig for oscillating flow

### Uncertainty analysis

Uncertainty in the experimental data is considered by identifying the main sources of errors in the primary measurements such as power supplied, temperature, time and frequency. Then, an uncertainty analysis based on the method described by Figliola and Beasley [11] has been performed. The uncertainties of dimensions, power, temperature, and Nusselt numbers are estimated to be 0.31%, 0.66%, 0.67%, and 2.39%, respectively.

### Results and discussion

Heat transfer experiments for estimating the Nusselt number in oscillating flow have been conducted at three different power inputs  $q$ , by selecting three different piston strokes,  $x_{p\text{-max}}$  and three different frequency outputs of A/C drive  $n$ . This way 27 sets of experiments have been conducted. Rotational speed of moto-reductor at 50 Hz line frequency is nominally 70 rpm. Thus rotational speed and corresponding frequency are calculated by  $N = (70/50)n$  and  $f = N/60$ , respectively, for a given drive frequency  $n$ . Angular frequency is  $\omega = 2\pi f$ . Actual rotational speed of the moto-reducer determined is to be 69.9 rpm using FFT (Matlab©). The displacement of the piston has been taken as zero at the rear position inside the cylinder and it becomes a maximum which is equal to the diameter of the flywheel at the forward position. The piston displacement is equal to the fluid displacement due to the fact that the fluid is incompressible, as water has been used as flow medium. Hence, at the entrance of the heat exchangers where heat transfer calculations have been made, the fluid displacement  $x_m$  varies according to:

$$x_m(t) = \frac{x_{\text{max}}}{2}(1 - \cos \omega t) \quad (1)$$

where  $x_{\text{max}} = x_{p\text{-max}} A_p / A = 2RA_p / A$ . Here,  $R$ ,  $A_p$ , and  $A$  are flywheel radius, cross-sectional areas of double acting cylinder and inner pipe of heat exchanger, respectively. The cross-sectional mean fluid velocity in the pipe is  $u_m(t) = u_{\text{max}} \sin \omega t$  where  $u_{\text{max}} = \omega x_{\text{max}} / 2$ .

Time averaging for temperature data has been made for 60 cycles. Temperature variation at the heat exchanger inlet has been assumed to be sinusoidal in the form  $T(t) = T_m + T_{\text{max}} \sin \omega t$  where  $T_m$  is the time average of the temperature and  $T_{\text{max}}$  is the amplitude of the temperature variation. The assumption has been verified by comparing the results of this assumption with

the experimental data, fig. 2. This is the test run for 130 mm stroke length, 300 W power input and 10 Hz line frequency.

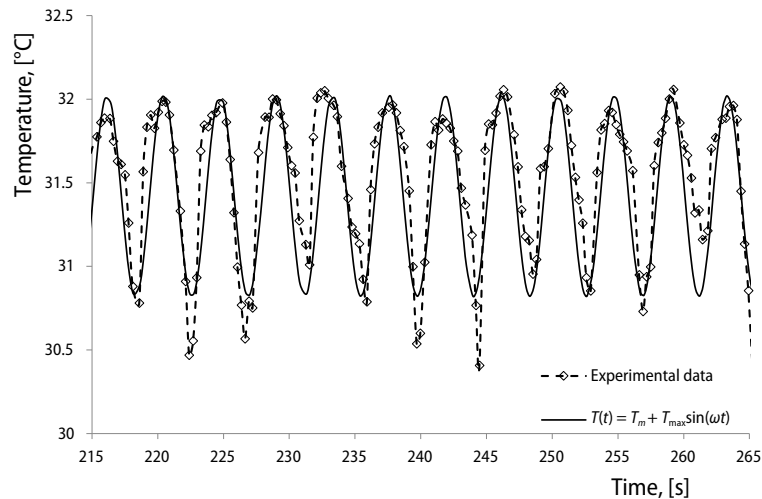


Figure 2. Variation of temperature of oscillating fluid at the heat exchanger inlet

Heat balance calculations for heat exchangers have been performed as shown in tab. 1. Temperature difference at the inlet and outlet and the flowrate of cooling water supplied by the constant temperature bathes, have been used to perform the calculations in the form  $Q_{absorbed} = mC_p\Delta T$  where  $m$  is the mass flow rate of cooling water from the CTB, and  $\Delta T$  is the temperature difference between the delivered and returned cooling water. Heat rate applied at the heated section has been calculated in the form  $Q_{applied} = IV$  where  $I$  and  $V$  are current in amperes and voltage in volts, respectively. It has been assumed that two symmetrical heat exchangers share the heat load equally, thus each absorbing half the applied heat. As can be seen from the table, both measured (applied) heat rate and absorbed heat rate are very close to each other which means uncertainty in power measurements are actually very low. The precision of data collected helps construct a reliable Nusselt number correlation. These calculated heat rate values have been used to calculate the Nusselt numbers.

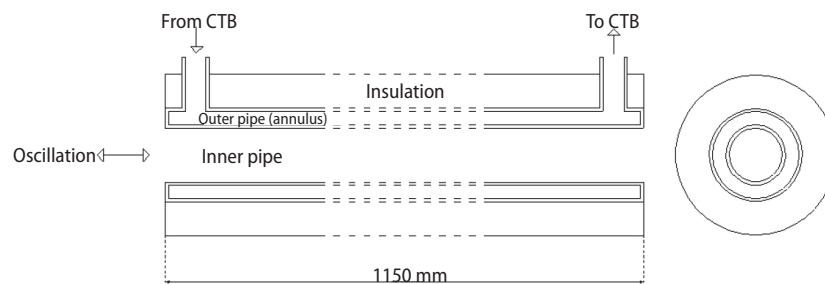
Heat transfer calculations for heat exchangers have been performed solving the radial heat transfer problem in an assembly made up of two concentric pipes whose outer surface is insulated. However, due to the fact that the environmental temperature and the cooling water temperature are almost the same (20 °C), insulation actually has no importance. Figure 3 shows the domain where the heat transfer calculations have been made. Heat transfer for the oscillating flow coefficient has been constructed in the following manner:

$$\frac{Q}{\pi dL} = Q'' = h_{in}(T_w - T_m) \quad (2)$$

where  $T_w$  is the time averaged temperature inner wall temperature of the inner pipe of the heat exchanger and  $T_m$  is the time averaged temperature of the oscillating flow at the heat exchanger inlet. Prandtl numbers and other water properties have been calculated as functions of temperature for each experimental run according to the polynomials given by Dixon [12].

**Table 1. Heat balance calculation for heat exchangers**

Experiment	Stroke, [m]	N, [rpm]	$Q_{applied}$ , [W]	$Q_{absorbed}$ , [W]	Difference, [%]
1	0.130	5	300	148.8	0.8%
2	0.130	10	300	147.8	1.5%
3	0.130	15	300	151.3	-0.9%
4	0.130	5	400	196.5	1.7%
5	0.130	10	400	198.1	1.0%
6	0.130	15	400	197.5	1.3%
7	0.130	5	500	244.8	2.1%
8	0.130	10	500	250.9	-0.4%
9	0.130	15	500	248.0	0.8%
10	0.170	5	300	150.7	-0.5%
11	0.170	10	300	146.6	2.3%
12	0.170	15	300	149.8	0.1%
13	0.170	5	400	199.3	0.3%
14	0.170	10	400	201.1	-0.5%
15	0.170	15	400	202.0	-1.0%
16	0.170	5	500	248.1	0.7%
17	0.170	10	500	251.4	-0.6%
18	0.170	15	500	248.8	0.5%
19	0.195	5	300	148.6	1.0%
20	0.195	10	300	146.9	2.1%
21	0.195	15	300	151.4	-0.9%
22	0.195	5	400	201.7	-0.9%
23	0.195	10	400	201.6	-0.8%
24	0.195	15	400	195.8	2.1%
25	0.195	5	500	252.8	-1.1%
26	0.195	10	500	252.7	-1.1%
27	0.195	15	500	246.6	1.4%



**Figure 3. Schematic of heat exchanger**

Because the heat exchanger is a *parallel flow* one in half cycle whereas it is *counter flow* in the other half, it is necessary to calculate the logarithmic mean temperature differences (LMTD) separately for each half of the cycle. However, it turns out that the arithmetic differences and logarithmic mean temperatures are basically same and the difference is negligible due to the fact that inlet-outlet temperature differences of heated and cooled fluids are small.

Nusselt number for unidirectional laminar annulus flow has been calculated using:

$$\text{Nu}_{annulus} = \frac{h_{annulus} D_h}{k_w} = 7.54 + \frac{0.03 \text{Re} \text{Pr} \frac{D_h}{L}}{1 + 0.016 \left[ \text{Re} \text{Pr} \frac{D_h}{L} \right]^{2/3}} \quad (3)$$

as given by Staton and Cavagnino [13] where  $D_h$  is the hydraulic diameter of annulus. Once the heat transferred is determined, radial heat transfer problem is solved for each experimental run to calculate the thermal resistance  $\Sigma R = \Delta T/Q$  where:

$$\Sigma R = \frac{1}{h_{in} \pi d L} + \frac{\ln \frac{r_{out}}{r}}{2 \pi L k_s} + \frac{1}{h_{annulus} \pi d_{out} L} \quad (4)$$

Heat transfer coefficients  $h_{in}$  obtained above have been used to calculate Nusselt number for each run:

$$\text{Nu} = \frac{h_{in} d}{k_w} \quad (5)$$

Nusselt numbers obtained above have been plotted against  $\text{Re}_\omega$  for each non-dimensional  $A_o$  as shown in fig. 4 where  $A_o$  is the non-dimensional displacement length defined as  $A_o = x_{\max}/d$ , and  $\text{Re}_\omega$  is the kinetic Reynolds number defined as  $\text{Re}_\omega = \rho \omega d^2/\mu$ .

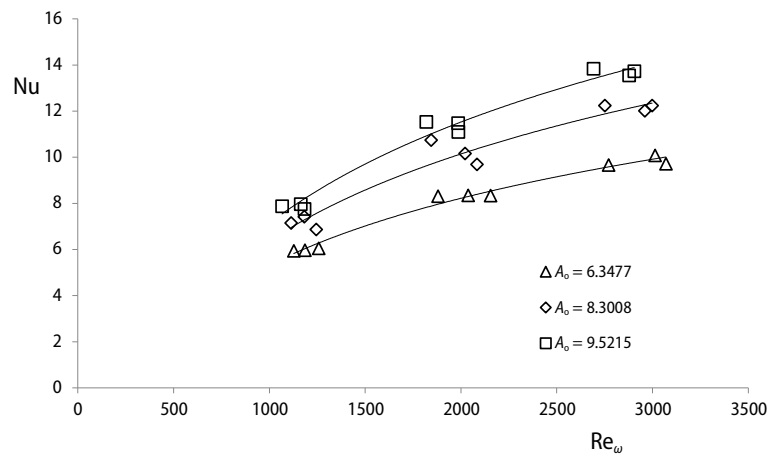


Figure 4. Variation of Nusselt number with kinetic Reynolds number

Having obtained all the Nusselt numbers using 27 sets of experimental data, a non-linear correlation has been obtained utilizing IBM SPSS© non-linear regression module in the form  $\text{Nu} = a A_o^b \text{Re}_\omega^c \text{Pr}^d$  with the following parameters:  $a = 0.027$ ,  $b = 0.752$ ,  $c = 0.443$ , and  $d = 0.603$ .

Thus the new correlation is given:

$$\text{Nu} = 0.027 A_o^{0.752} \text{Re}_\omega^{0.443} \text{Pr}^{0.603} \quad (6)$$

This correlation is valid for  $6.3477 < A_o < 9.5215$  and  $1066 < Re_\omega < 3070$ .

The works of previous researches such as Hausen's equation (Fan *et al.*, [14]), Zhao and Cheng [9] to be compared with the new correlation are as follows:

– Hausen's equation for cooled section:

$$Nu = 3.3 Wo^{0.2} + \frac{0.041 \frac{A_d}{L_c} Wo^2 Pr}{1 + 0.016 \left[ \frac{A_d}{L_c} Wo^2 Pr \right]^{2/3}} \quad (7)$$

where  $A_d$  is the fluid displacement and  $L_c$  is the length of the cooled section.

– Zhao and Cheng correlation:

$$Nu = 0.02 A_o^{0.85} Re_\omega^{0.58} \quad (8)$$

Figure 5 compares the experimental results with the new correlation above and with that of other researchers in the form  $Re_\omega$  vs.  $Nu/A_o^{0.752}$ .

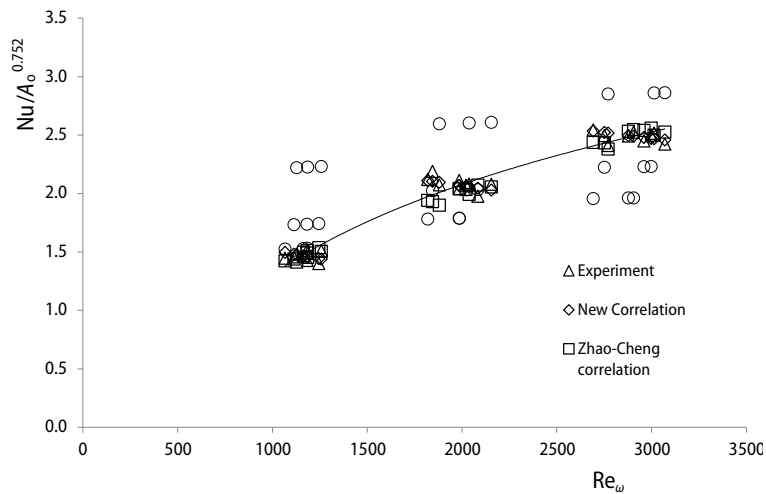


Figure 5. Comparison of experimental  $Nu/A_o^{0.752}$  with various correlations

## Conclusion

In this experimental study, heat transfer in oscillating flow where water is the flow medium has been studied. A new Nusselt number correlation has been obtained for oscillating flow and compared to those found in the literature. Because this new correlation has been obtained conducting a wide range of experiments and it takes into account the effect of Prandtl number which considerably changes with temperature variations, it is considered to be a reliable tool to estimate the heat transfer aspects of an oscillating flow.

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## Nomenclature

$A$	– cross-sectional area of concentric heat exchanger's inner pipe	$Re_{\omega}$	– kinetic Reynolds number ( $= \rho \omega d^2 / \mu$ ), [–]
$A_o$	– non-dimensional displacement, ( $= x_{max} / d = 2Re_d / Re_{\omega}$ ), [–]	$T$	– temperature, [°C]
$A_p$	– cross-sectional area of double acting cylinder	$t$	– time, [s]
$C_p$	– constant pressure specific heat	$u_m$	– cross-sectional mean fluid velocity, [ms <sup>-1</sup> ]
$D$	– inner diameter diameter of concentric heat exchanger's outer pipe	$u_{max}$	– amplitude of mean fluid velocity, [ms <sup>-1</sup> ]
$d$	– inner diameter diameter of concentric heat exchanger's inner pipe	$Wo$	– Womersley number ( $= 0.5(Re_{\omega})^{1/2}$ ), [–]
$h$	– heat transfer coefficient	$x_m$	– temporal fluid displacement at the inlet of the heat exchanger
$k$	– conduction coefficient	$x_{max}$	– maximum fluid displacement in concentric heat exchanger's inner pipe
$L$	– length of the concentric heat exchanger	$x_p$	– maximum displacement of the piston (stroke)
$N$	– rotational speed of motor-reducer, [rpm]	<i>Greek symbols</i>	
$Nu_L$	– space-cycle averaged Nusselt number ( $= hd/k_w$ ), [–]	$\mu$	– dynamic viscosity, [Pas]
$n$	– frequency output of A/C drive, [Hz]	$\nu$	– kinematic viscosity, [m <sup>2</sup> s <sup>-1</sup> ]
$Q$	– joule heating obtained from ribbon heaters, [W]	$\rho$	– density, [kgm <sup>-3</sup> ]
$Q''$	– heat flux at the wall, [Wm <sup>-2</sup> ]	$\omega$	– angular frequency
$R$	– radius of flywheel	<i>Subscripts</i>	
$Re_d$	– pipe diameter based Reynolds number ( $= \rho u_m d / \mu$ ), [–]	p	– piston
		s	– solid
		w	– water

## References

- [1] Akdag, U., *et al.*, The Experimental and Theoretical Investigation of Heat Transfer in Oscillating Flow by Control Volume Approach, *Journal of Thermal Science and Technology*, 28 (2000), 1, pp. 23-31
- [2] Ozdemir, M., Ozguc, A. F., A Simple Mathematical Model to Analyse a Fluidyne Heat Machine, *Proc. Instn Mech. Engrs, Part A: Journal of Power and Energy*, 217 (2003), 1, pp 91-100
- [3] Akdag, U., Ozdemir, M., Heat Transfer in an Oscillating Vertical Annular Liquid Column Open to Atmosphere, *Heat Mass Transfer*, 42 (2006), 7, pp. 617-624
- [4] Arslan, G., Ozdemir, M., Correlation to Predict Heat Transfer of an Oscillating Loop Heat Pipe Consisting of Three Interconnected Columns, *Energy Conversion and Management*, 49 (2008), 8, pp. 2337-2344
- [5] Akdag, U., *et al.*, Heat Removal from Oscillating Flow in a Vertical Annular Channel, *Heat Mass Transfer*, 44 (2008), 4, pp. 393-400
- [6] Zhao, T. S., Cheng, P., Heat Transfer in Oscillatory Flows, *Annual Review of Heat Transfer*, Vol. IX, 1998, pp. 359-420
- [7] Shahin, G. A., The Effect of Pulsating Flow on Forced Convective Heat Transfer. (Thesis Study), Faculty of Graduate Studies, The University of Western Ontario, London, Ont., Canada, 1998
- [8] Walther, C., *et al.*, Numerical Investigations on the Heat Transfer in Turbulent Oscillating Pipe Flow, *Heat and Mass Transfer*, 36 (2000), 2, pp. 135-141
- [9] Zhao, T., Cheng, P., Oscillatory Heat Transfer in a Pipe Subjected to a Laminar Reciprocating Flow, *ASME Journal of Heat Transfer*, 118 (1996), 3, pp. 592-598
- [10] Bouvier, P., *et al.*, Experimental Study of Heat Transfer in Oscillating Flow, *International Journal of Heat and Mass Transfer*, 48 (2005), 12, pp. 2473-2482
- [11] Figliola, R. S., Beasley, D. E., *Theory and Design for Mechanical Measurements*, John Wiley, New York, USA, 2006
- [12] Dixon, J., *The Shock Absorber Handbook*, 2<sup>nd</sup> ed., Annex C, John Wiley, ISBN: 978-0-470-51020-9, 2007
- [13] Staton, D. A., Cavagnino, A., Convection Heat Transfer and Flow Calculations Suitable for Electric Machines Thermal Models, *IEEE Transactions of Industrial Electronics*, 55 (2008), 10, pp. 3509-3516
- [14] Fan, A., *et al.*, Experimental Study of Oscillating Flow Heat Transfer, *Proceedings, NHT2008 Micro/Nanoscale Heat Transfer International Conference*, Tainan, Taiwan, 2008, pp. 347-354