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ORIGINAL ARTICLE

Re-examining the real interest rate parity hypothesis under temporary gradual breaks and nonlinear convergence

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Abstract
This paper investigates the real interest parity hypothesis by testing stationarity of real interest rate differentials for 52 countries with respect to the USA. Taking account of the fact that both asymmetric adjustment and gradual temporary breaks may better characterize the dynamics of real interest rate differentials, we propose a new test that allows for two temporary shifts together with asymmetric adjustment towards the equilibrium. We employ the newly proposed test procedure along with the conventional ADF test as well as nonlinear KSS and OSH tests to examine stationarity of real interest rate differentials. Among the main results, we find that the newly proposed unit root test procedure highly outperforms the existing unit root tests in terms of rejecting the null hypothesis of unit root. Our results suggest that real interest rate differentials can be characterized by a stationary process with asymmetric adjustment around gradual and temporary shifts of mean.

Keywords RIRP · Multiple smooth breaks · ESTR trend · ESTAR nonlinearity

JEL Classification C12 · C22 · F36 · F40

1 Introduction

Free movement of goods and capital across borders has long been viewed by economists as a prerequisite for economic efficiency. Therefore, many countries have been implementing massive reforms to liberalize their domestic markets and

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to increase integration to the world economy to bolster economic development. The degree of integration of domestic markets to the world economy can easily be assessed by testing empirical fulfilment of the real interest rate parity (Frankel 1992; Chinn and Frankel 1995; Obstfeld and Taylor 2002; Lothian 2002). The real interest rate parity (RIRP) holds that if agents form their expectations rationally and arbitrage forces eliminate profitable trade opportunities in both goods and assets markets, then the real interest rates will tend to equalize across countries. Hence, empirical fulfilment of the RIRP is considered as an evidence of integration of markets between countries.

Whether the RIRP hypothesis holds or not has important implications for economists, policy authorities and market practitioners. The RIRP hypothesis is considered to be a building block of many exchange rate and open-economy macroeconomics models as these models are usually built on the assumption of equality of prices and interest rates across countries (e.g., Dornbusch 1976; Mussa 1976). From the policy perspective, the validity of RIRP implies that monetary authorities' influence on real macroeconomic variables through the interest rate channel are restricted, since the domestic real interest rate will be determined by the world real interest rate (Phylaktis 1999; Ferreira and Leon-Ledesma 2007). On the other hand, the violation of RIRP is a necessary condition for policy makers to influence the economy through the real interest rate channel (Mark 1985). Violation of the RIRP hypothesis points to profitable trading opportunities in goods and assets markets for market practitioners.

Given the importance of the RIRP hypothesis, examination of the empirical fulfilment of the RIRP hypothesis has attracted huge interest of empirical researchers. Early studies that used conventional regression methods or cointegration tests presented limited or no evidence of fulfilment of the RIRP hypothesis (e.g., Mishkin 1984; Cumby and Obstfeld 1984; Meese and Rogoff 1988; Cavaglia 1992; Chinn and Frankel 1995; Crowder 1995). Being unsatisfied with results contradicting the RIRP hypothesis, researchers have increasingly been searching for alternative methods to test the RIRP hypothesis. Some researchers proposed using panel methods to improve power of the conventional tests in small samples (e.g., Wu and Chen 1998; Holmes 2002; Baharumshah et al. 2005; Albulescu et al. 2016). Other researchers pointed to possible nonlinearities in financial variables, and used techniques compatible with nonlinear dynamics (i.e.,Cuestas and Harrison 2010; Öge Güney and Hasanov 2014; Çorakcı et al. 2017). Nonlinearity in real interest rate dynamics may arise because of various factors such as the presence of transaction costs, price rigidities, asymmetric information, interaction of agents with heterogeneous beliefs, official interventions into financial markets, etc.

Another strand of the literature used methods allowing for structural breaks in series under investigation (Ferreira and Leon-Ledesma 2007; Camarero et al. 2010; Baharumshah et al. 2013) arguing that abnormal political changes, regime changes in monetary policy, technological shocks, demand shocks, financial reforms and economic crises may lead to structural changes in many economic variables. Both strands of the literature provided evidence in favour of the RIRP hypothesis using methods allowing for structural breaks or nonlinearities.

Economists now acknowledge that many economic variables are best characterized by both structural breaks and nonlinear dynamics (e.g., Lundbergh et al. 2003). Therefore, we test the empirical validity of the RIRP hypothesis by taking account of both structural changes and nonlinearities. Specifically, to test the RIRP hypothesis, we examine stochastic properties of the real interest rate differentials (RID) between countries. While there may be differences between real interest rates because of market rigidities, these differences must be temporary in nature and real interest rate series must tend to equalize across countries in order the RIRP hypothesis to hold. This implies that the RIDs are a mean-reverting process under the RIRP hypothesis.[1] Therefore, stationarity of RIDs are considered as empirical evidence in favour of the RIRP hypothesis. On the other hand, presence of a unit root implies that deviations between real interest rates are persistent in nature, contradicting the RIRP hypothesis.

In this paper, we contribute to the literature in two different ways. First, we propose a new unit root test procedure that allows for two temporary shifts in the deterministic component of the series along with asymmetric and nonlinear adjustment towards the equilibrium. Previous tests of the RIRP that allowed for a break in the series have used test methods which assume that structural breaks are permanent or periodic (e.g., Gulcu and Yıldırım 2019). However, the RID does not return to pre-break level if the break is permanent. Permanent shifts in the RIDs violate the purchasing power parity (PPP) proposition, upon which the RIRP hypothesis is built. Also, continuous periodic fluctuations imply predictable pattern for the mean of the series, which violates no-arbitrage condition (see also discussions in Omay et al. 2020). Therefore, we model shifts in the deterministic components of the series via exponential smooth transition (ESTR) functions that restrict breaks to be temporary. The adjustment towards equilibrium is modelled via asymmetric ESTR function which allows the speed of adjustment towards equilibrium to depend on both the sign and size of disequilibrium. The use of ESTR functions allows both the shifts and adjustments to equilibrium to be gradual rather than abrupt. Another nice feature of using the ESTR functions is that this modelling approach allows a no break, a single break and a linear adjustment as special cases. Thus, the test procedure proposed in this paper provides a framework for testing stationarity of time series in line with theoretically consistent dynamics of many economic and financial variables. In fact, it is widely agreed that many economic and financial variables exhibit some form of nonlinearities (e.g., Terasvirta et al. 2010). On the other hand, real and nominal shocks as well as policy interventions may lead to shifts in the equilibrium level of many economic and financial variables. However, the shifts in some variables (e.g., real exchange rates, real interest rates) must be temporary in nature to ensure equilibrium in goods and financial markets. Also, some shocks or policy interventions might be transitory in nature (e.g., the coronavirus pandemic, full deposit insurances or FX-protected deposit schemes), which will cause only to a temporary shift in the level or trend of economic and financial variables. Thus, the test procedure can be used to test stationarity of many financial and economic variables consistent with underlying economic theory.

[1] Formally, the RIRP hypothesis implies that RID series are zero-mean white-noise process. However, transaction costs and/or varying risk premiums across countries may lead to non-zero mean RID series.

Second, the paper contributes to the RIRP literature by examining stationarity of the real interest rate differentials of 52 countries. In this paper, we have covered all countries for which we could obtain relevant data on price indices and interest rates. In addition to the tests proposed in this paper, we also employed the conventional ADF (augmented Dickey-Fuller (1979) test as well as the nonlinear unit root tests of Kapetanios et al. (2003) (the KSS test henceforth) and Omay et al. (2020) (the OSH test). While the KSS test allows for nonlinear adjustment towards equilibrium but no breaks, the OHS test allows both a shift in the deterministic component of the series and nonlinearities in the adjustment towards equilibrium. Applying various tests helps to shed light on stochastic properties of the RID series and explain limited empirical support for the RIRP hypothesis. Our findings imply that allowing more complex dynamics results in more empirical support for the RIRP hypothesis. In particular, we find that the test proposed in this paper outperforms all these tests in terms of rejection of the null hypothesis of unit root, consistent with the RIRP hypothesis. In addition, we find that allowing for a single break and nonlinearity provides more support for the RIRP hypothesis when compared to the case allowing for nonlinearity only, which, in turn, provides more support than the linear model. These findings also help to explain the mixed results in the literature. Overall, our results provide strong evidence in favour of the RIRP hypothesis and point to importance of taking account of complex dynamics in analysing financial data.

The rest of the paper is organized as follows. In the next Section 2 we briefly discuss the RIRP hypothesis. Section 3 presents the unit root test procedure, derive critical values and examine small sample performance of the newly proposed test. Test results are presented in Section 4. Finally, Section 5 concludes.

2 Theoretical background: The RIRP hypothesis

The RIRP hypothesis is based on the uncovered interest parity (UIP), purchasing power parity (PPP), Fisher hypothesis, and rational expectations. The UIP holds that the expected change in the exchange rate is exactly equal to the interest rate difference between domestic and foreign interest rates.

$$\Delta s_t^e = i_t - i_t^* \tag{1}$$

where Δs_t^e is the expected change in the exchange rate, i_t and i_t^* are the domestic and foreign interest rates. The PPP emphasize that exchange rate change is equal to the inflation differentials between countries.

$$\Delta s_t = \pi_t - \pi_t^* \tag{2}$$

where Δs_t is the exchange rate change, π_t and π_t^* are the domestic and foreign inflation rates. Finally, rational expectation hypothesis is represented by the following equation:

$$\Delta s_t^e = \Delta s_t + \varepsilon_t \tag{3}$$

Substituting eq. (3) and then eq. (2) into eq. (1) yields

$$i_t - i_t^* = \pi_t - \pi_t^* + \varepsilon_t \tag{4}$$

Equation (4) can be rewritten as follows:

$$i_t - \pi_t = i_t^* - \pi_t^* + \varepsilon_t$$

According to the Fisher equation, $r_t = i_t - \pi_t$ and $r_t^* = i_t^* - \pi_t^*$ which define real interest rate as nominal interest rate less inflation rate over the same period. Finally, the real interest parity relationship is defined as follows:

$$RID = r_t - r_t^* = \varepsilon_t \tag{5}$$

According to the eq. (5), if RIRP hypothesis holds, the RID series is a zero mean stationary process. Hence, we can employ unit root tests to examine the stationarity of the RID series to see whether the RIRP hypothesis holds in the long run.

3 The methodology

3.1 Modelling temporary shifts in the deterministic component

Consider the following data generating process (DGP):

$$y_t = \phi(t) + u_t \tag{6}$$

where $\phi(t)$ is the deterministic nonlinear trend function and u_t are the deviations from the trend. This representation of the DGP has a nice property that it allows for the possible shifts under both the null and the alternative hypotheses, without introducing any structures that are irrelevant under either (see, for example, Schmidt and Phillips 1992).[2]

We consider double exponential smooth transition functions to model temporary gradual changes in the deterministic components of the series. Below are the three alternative models for modelling the deterministic components:

Model A:

$$\phi(t) = \alpha_1 + \alpha_2 S_1(\gamma_1, \tau_1) + \alpha_3 S_2(\gamma_2, \tau_2) \tag{7a}$$

Model B:

$$\phi(t) = \alpha_1 + \beta_1 t + \alpha_2 S_1(\gamma_1, \tau_1) + \alpha_3 S_2(\gamma_2, \tau_2) \tag{7b}$$

Model C:

[2] Omay et al. (2018) also used the same DGP as in eq. (6). But they used Fourier and logistic transition functions for the $\phi(t)$ trend function to model gradual changes in the deterministic components of the series.

$$\phi(t) = \alpha_1 + \beta_1 t + \alpha_2 S_1(\gamma_1, \tau_1) + \beta_2 t S_1(\gamma_1, \tau_1) + \alpha_3 S_2(\gamma_2, \tau_2) + \beta_3 t S_2(\gamma_2, \tau_2) \tag{7c}$$

with the exponential smooth transition function

$$S_i(\gamma_i, \tau_i) = 1 - \exp\left[-\gamma_i (t - \tau_i T)^2\right],\ \gamma_i > 0 \text{ for } i = 1, 2 \tag{8}$$

The transition function, $S_i(\gamma_i, \tau_i)$ is continuous, bounded between zero and one, and symmetric around zero. The transition parameters γ_i determine the smoothness of transitions such that the speed of structural shifts increase with γ_i. The parameters τ_i determine the timing of the transition midpoints such that $S_i(\gamma_i, \tau_i) = 0$ when $t = \tau_i T$ and $S_i(\gamma_i, \tau_i)$ approaches 1 when t moves further from $\tau_i T$; that is, $S_i(\gamma_i, \tau_i) = 1$ both for $t < \tau_i T$ and $t > \tau_i T$. These features of the transition function imply that the structural changes in the deterministic components of the series are temporary. Notice that the Model A is appropriate for modelling changes in the mean of a non-trending series whereas Models B and C can be used for modelling changes in the mean only and changes both in the mean and trend of series, respectively. The use of smooth transition functions has another nice feature that by appropriate parameter restrictions one obtains a single shift ($\gamma_2 = 0$) and no shift ($\gamma_1 = \gamma_2 = 0$) models as well. In fact, when the transition parameter $\gamma_i = 0$, the ESTR function collapses to zero, and hence one obtains a linear trend function.

In order to depict the nature of the changes that the ESTR functions can imitate, we generated series for different transition parameters (γ_i, τ_i) as well as changes parameters (α_i) using eq. (7a) above. The graphs of these generated series are presented below in Fig. 1. As can be seen from Fig. 1, the use of two ESTR functions $S_i(\gamma_i, \tau_i)$ can imitate pretty rich variety of changes observed in many financial variables.

3.2 Modelling adjustment to equilibrium

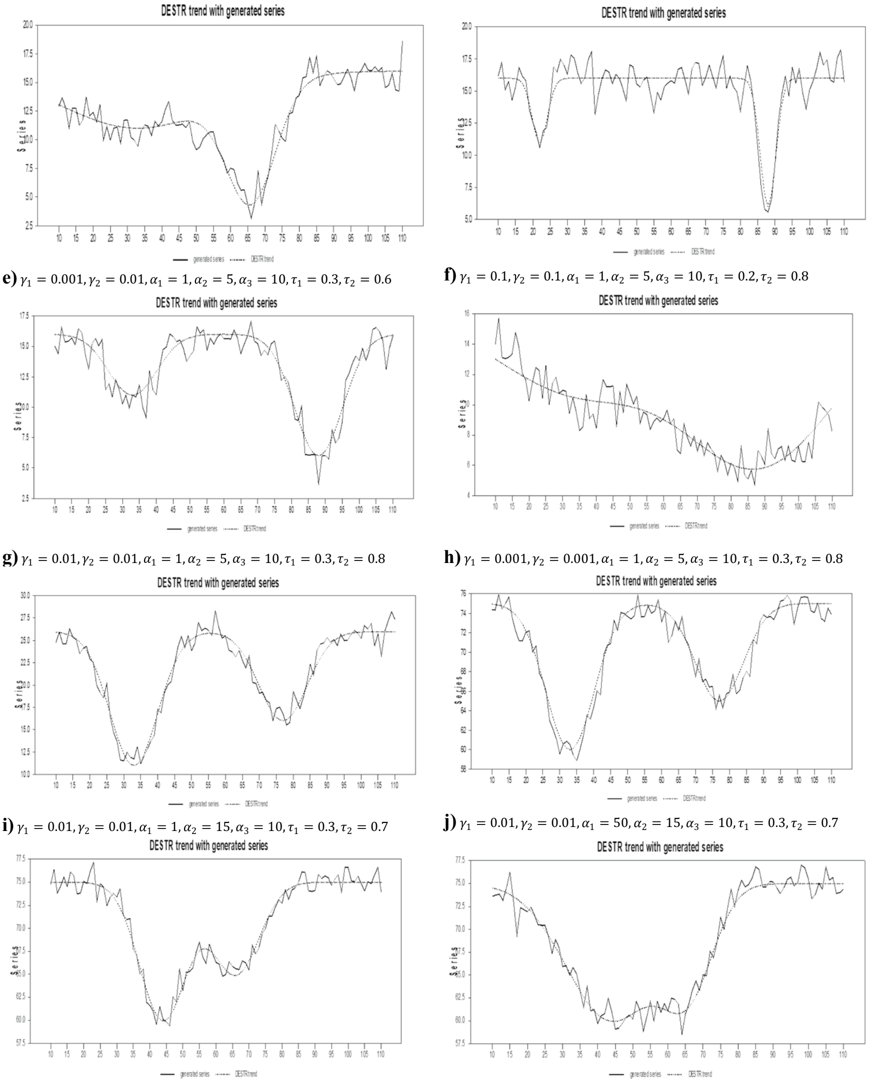
Following Sollis (2009) and Omay et al. (2018), we model deviations from the gradually changing trend function using the asymmetric exponential smooth transition functions. In particular, we consider the following asymmetric exponential smooth transition autoregressive (AESTAR) model for the deviations u_t:

$$\Delta u_t = G(\theta_1, u_{t-1})\left\{F(\theta_2, u_{t-1})\rho_1 + \left(1 - F(\theta_2, u_{t-1})\right)\rho_2\right\}u_{t-1} + \epsilon_t \tag{9}$$

$$G(\theta_1, u_{t-1}) = 1 - \exp\left(-\theta_1 \left(u_{t-1}^2\right)\right), \theta_1 > 0 \tag{10}$$

$$F(\theta_2, u_{t-1}) = \left[1 + \exp\left(-\theta_2 u_{t-1}\right)\right]^{-1}, \theta_2 > 0 \tag{11}$$

where $\epsilon_t \sim \text{i.i.d}(0, \sigma^2)$. The transition function $G(\theta_1, u_{t-1})$ is similar to $S(\gamma_i, \tau_i)$ given in eq. (8) above and the extreme values of this function are associated with the size of the deviations from the equilibrium. Notice that $G(\theta_1, u_{t-1}) \to 0$ as $u_{t-1} \to 0$ and $G(\theta_1, u_{t-1}) \to 1$ as $u_{t-1} \to \pm\infty$, implying that the deviations u_t may be highly



e) $\gamma_1 = 0.001, \gamma_2 = 0.01, \alpha_1 = 1, \alpha_2 = 5, \alpha_3 = 10, \tau_1 = 0.3, \tau_2 = 0.6$

f) $\gamma_1 = 0.1, \gamma_2 = 0.1, \alpha_1 = 1, \alpha_2 = 5, \alpha_3 = 10, \tau_1 = 0.2, \tau_2 = 0.8$

g) $\gamma_1 = 0.01, \gamma_2 = 0.01, \alpha_1 = 1, \alpha_2 = 5, \alpha_3 = 10, \tau_1 = 0.3, \tau_2 = 0.8$

h) $\gamma_1 = 0.001, \gamma_2 = 0.001, \alpha_1 = 1, \alpha_2 = 5, \alpha_3 = 10, \tau_1 = 0.3, \tau_2 = 0.8$

i) $\gamma_1 = 0.01, \gamma_2 = 0.01, \alpha_1 = 1, \alpha_2 = 15, \alpha_3 = 10, \tau_1 = 0.3, \tau_2 = 0.7$

j) $\gamma_1 = 0.01, \gamma_2 = 0.01, \alpha_1 = 50, \alpha_2 = 15, \alpha_3 = 10, \tau_1 = 0.3, \tau_2 = 0.7$

k) $\gamma_1 = 0.002, \gamma_2 = 0.01, \alpha_1 = 50, \alpha_2 = 15, \alpha_3 = 10, \tau_1 = 0.3, \tau_2 = 0.7$

l) $\gamma_1 = 0.003, \gamma_2 = 0.01, \alpha_1 = 1, \alpha_2 = 15, \alpha_3 = 10, \tau_1 = 0.3, \tau_2 = 0.7$

Fig. 1 Simulated series with two temporary gradual changes in the mean

persistent for small values whereas are quickly corrected for larger values. The transition function $F(\theta_2, u_{t-1})$ is also restricted between 0 and 1 whereas these regimes are associated with negative and positive values of deviations from the equilibrium as u_t is a zero-mean process. In particular, $F(\theta_2, u_{t-1}) \rightarrow 0$ as $u_{t-1} \rightarrow -\infty$ and $F(\theta_2, u_{t-1}) \rightarrow 1$ as $u_{t-1} \rightarrow +\infty$.

To see how the adjustment of the deviations are governed by the AESTAR model given above, first consider the case when $u_{t-1} \to -\infty$. In this case, the transition function $F(\theta_2, u_{t-1}) \to 0$ and therefore eq. (9) collapses to

$$\Delta u_t = G(\theta_1, u_{t-1})\rho_2 u_{t-1} + \epsilon_t \tag{12}$$

Hence, since $G(\theta_1, u_{t-1})$ moves from zero to one as $u_{t-1} \to -\infty$, the model implies transition from the inner regime

$$\Delta u_t = \epsilon_t \tag{13}$$

to the outer regime

$$\Delta u_t = \rho_2 u_{t-1} + \epsilon_t \tag{14}$$

associated with the extreme values of the transition function $G(\theta_1, u_{t-1})$.

On the other hand, the transition function $F(\theta_2, u_{t-1}) \to 1$ as $u_{t-1} \to +\infty$, resulting in:

$$\Delta u_t = G(\theta_1, u_{t-1})\rho_1 u_{t-1} + \epsilon_t \tag{15}$$

In this case, the transition will be from the inner regime

$$\Delta u_t = \epsilon_t$$

to the outer regime

$$\Delta u_t = \rho_1 u_{t-1} + \epsilon_t \tag{16}$$

as the exponential function $G(\theta_1, u_{t-1})$ moves from zero to one when $u_{t-1} \to +\infty$.

Global stationarity of the AESTAR process depends on the conditions $\theta_1 > 0$, $\rho_1 < 0$,$\rho_2 < 0$ (see also, Sollis 2009). A nice feature of the AESTAR model is that it nests the symmetric exponential smooth transition autoregressive (ESTAR)-type adjustment towards equilibrium as in Kapetanios et al. (2003). In particular, when $\rho_1 = \rho_2$ or $\theta_2 = 0$ one obtains ESTAR model (see, for example, Omay et al. 2020). In this case, the adjustment towards equilibrium will depend on the size of deviations, irrespective of the sign. This implies that the adjustment is governed by the following

$$\Delta u_t = G(\theta, u_{t-1})\rho u_{t-1} + \epsilon_t \tag{17}$$

instead of eq. (9). The ESTAR process given in eq. (17) implies that while the deviations may follow a unit root process around nonlinear attractor (given in eqs. 7a-7c) the process is globally stationary around the attractor if $\rho < 0$. In fact, notice that $G(\theta, u_{t-1}) \to 1$ as $u_{t-1} \to +\infty$, in which case eq. (17) collapses to:

$$\Delta u_t = \rho u_{t-1} + \epsilon_t \tag{18}$$

which is a stationary process provided that $\rho < 0$.

3.3 Testing the null hypothesis of unit root

As briefly discussed above, the AESTAR model given in eq. (9) collapses to the unit root process given in eq. (13) when $G\left(\theta_1, u_{t-1}\right) = 0$. Therefore, the null hypothesis of unit root can be tested against the alternative of AESTAR nonlinearity by testing

$$H_0 : \theta_1 = 0 \tag{19}$$

against the alternative:

$$H_1 : \theta_1 > 0 \tag{20}$$

However, this is not feasible as the parameters θ_2, ρ_1 and ρ_2 are not identified under this null hypothesis. The problem of unidentified nuisance parameters under the null can be circumvented by replacing the transition functions by appropriate Taylor series approximation following Luukkonen et al. (1988). Replacing the transition functions $G\left(\theta_1, u_{t-1}\right)$ and $F\left(\theta_2, u_{t-1}\right)$ by their first-order Taylor series approximations, and allowing for serial correlation in the disturbances ϵ_t we obtain:

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \varphi_2 u_{t-1}^4 + \sum_{j=1}^{p} \delta_j \Delta u_{t-j} + \vartheta_t \tag{21}$$

where φ_1 and φ_2 are functions of $\theta_1, \theta_2, \rho_1, \rho_2$, and the ϑ_t term comprises the original disturbances ϵ_t as well as the error term arising from the Taylor approximation. Now, the null hypothesis $H_0 : \theta_1 = 0$ can equally be tested by

$$H_0 : \varphi_1 = \varphi_2 = 0 \tag{22}$$

by the conventional F-statistics using regression eq. (21).

In the case of symmetric ESTAR-type nonlinear adjustment towards equilibrium as given in eq. (17) above, replacement of the transition function with its first-order Taylor approximation produces the following auxiliary regression equation:

$$\Delta u_t = \varphi u_{t-1}^3 + \sum\nolimits_{j=1}^{p} \delta_j \Delta u_{t-j} + \vartheta_t \tag{23}$$

Therefore, the null hypothesis of unit root can be tested against the ESTAR-type stationarity by the conventional t-test associated with φ in the regression eq. (23).

As in Kapetanios et al. (2003) and Leybourne et al. (1998), these tests can be carried out in two steps. In the first, one estimates the preferred deterministic component given in eqs. (7a-7c) and collects residuals $\hat{u}_t$. In the second step, using the de-trended series $\hat{u}_t$ one estimates the regression eq. (21) and/or (23) by ordinary least squares estimator and tests the null hypothesis using conventional F- and/or t-tests. We denote these test statistics as $F_{2AE\alpha}(t_{2E\alpha})$ if the model A is used, $F_{2AE\alpha(\beta)}$ $(t_{2E\alpha(\beta)})$ if the model B is used, and $F_{2AE\alpha\beta}(t_{2E\alpha\beta})$ if the model C is used to model the deterministic component of the series.

As the nonlinear least squares estimator of the transition parameters $(\gamma_1, \gamma_2, \tau_1, \tau_2)$ in eqs. (7a-7c) does not admit closed-from solutions of transition parameter we generated the critical values of test statistics via stochastic simulations. Simulated critical values are reported in Table 1.

3.4 Small sample properties of the proposed test

Performance of the proposed test statistics in small samples is analysed via small Monte Carlo simulations. Size properties of the tests are evaluated using two different DGPs, given below in eqs. (24) and (25)-(26)[3]:

$$y_t = y_{t-1} + \varepsilon_t \tag{24}$$

$$\varepsilon_t \sim iidN(0, 1)$$

which does not allow a break under the null hypothesis, and

$$y_t = \phi(t) + u_t \tag{25}$$

where

$$\phi(t) = \alpha_1 + \alpha_2 S_1(\gamma_1, \tau_1) + \alpha_3 S_2(\gamma_2, \tau_2) \tag{26a}$$

$$u_t = u_{t-1} + \varepsilon_t \tag{26b}$$

$$\varepsilon_t \sim iidN(0, 1)$$

which allows two temporary shifts in the mean of the series.

The results reported in Table 2 below shows that both the t- and F-statistics have quite satisfactory size properties.[4]

The power performance of the proposed tests in small samples was evaluated using the following DGP:

$$y_t = \alpha_1 + \alpha_2 S_1(\gamma_1, \tau_1) + \alpha_3 S_2(\gamma_2, \tau_2) + u_t \tag{27}$$

$$\Delta u_t = G(\theta_1, u_{t-1})\{F(\theta_2, u_{t-1})\rho_1 + (1 - F(\theta_2, u_{t-1}))\rho_2\}u_{t-1} + \varepsilon_t$$

[3] We would like to thank an anonymous reviewer for reminding us to analyse the size of the tests with break under the null hypothesis as well.

[4] In order to save space, we report simulation results only for the $t_{2E\alpha}$ and $F_{2AE\alpha}$ statistics (i.e., Model A). Size properties of the $t_{2E\alpha(\beta)}$, $F_{2AE\alpha(\beta)}$, $t_{2E\alpha\beta}$, $F_{2AE\alpha\beta}$ statistics (Models B and C) were quite similar to those reported in Table 2. Also, we analysed the size of other tests used in the empirical part of this paper, i.e., Omay et al. (2020) and Omay et al. (2018) under two temporary shifts. While the sizes of these tests were slightly higher than those of the $t_{2E\alpha}$ and $F_{2AE\alpha}$ tests, they were within the acceptable interval, i.e., did not exceed 7.5%. All the simulation results are available upon request from the corresponding author.

Table 1 Critical values of proposed test statistics

Model A

T	$t_{2E\alpha}$ 1%	5%	10%	$F_{2AE\alpha}$ 1%	5%	10%
25	-6.854	-5.006	-4.273	36.967	16.319	13.000
50	-6.760	-4.812	-4.162	33.424	16.047	12.844
100	-6.423	-4.743	-4.087	26.617	15.894	12.651
150	-6.399	-4.673	-4.023	26.465	15.834	12.598
200	-6.334	-4.603	-3.94	25.948	15.688	12.507
250	-6.236	-4.557	-3.894	25.317	15.626	12.371
300	-6.118	-4.500	-3.842	24.893	15.617	12.329
400	-5.949	-4.405	-3.732	24.566	15.260	12.152
500	-5.780	-4.277	-3.674	22.424	14.798	11.469
2000	-4.473	-3.884	-3.527	10.939	8.667	7.283

Model B

T	$t_{2E\alpha(\beta)}$ 1%	5%	10%	$F_{2AE\alpha(\beta)}$ 1%	5%	10%
25	-6.049	-5.233	-4.710	22.382	15.658	13.066
50	-5.379	-4.660	-4.239	16.774	11.968	10.089
100	-5.156	-4.490	-4.129	14.599	11.174	9.468
150	-5.149	-4.442	-4.106	14.152	10.871	9.350
200	-5.133	-4.430	-4.085	14.132	10.828	9.308
250	-5.061	-4.372	-4.068	14.022	10.712	9.202
300	-5.025	-4.365	-4.038	13.697	10.607	9.190
400	-4.974	-4.348	-4.016	13.476	10.574	9.141
500	-4.921	-4.297	-3.970	13.117	10.276	8.974
2000	-3.795	-2.982	-2.562	10.285	9.018	7.001

Model C

T	$t_{2E\alpha\beta}$ 1%	5%	10%	$F_{2AE\alpha\beta}$ 1%	5%	10%
25	-6.200	-5.263	-4.814	23.260	16.276	13.523
50	-5.596	-4.762	-4.319	16.549	12.498	10.498
100	-5.162	-4.464	-4.106	14.721	11.075	9.373
150	-5.085	-4.351	-4.030	14.010	10.659	9.143
200	-4.995	-4.330	-3.987	13.607	10.523	8.991
250	-4.981	-4.309	-3.965	13.446	10.329	8.912
300	-4.964	-4.298	-3.957	13.430	10.259	8.845
400	-4.916	-4.250	-3.917	13.241	10.101	8.660
2000	-4.723	-4.051	-3.718	11.973	9.439	8.031

Critical values are obtained by 10,000 replications.

Table 2 Size of the proposed tests

Panel A. Size of the tests under no break				
	T		$t_{2E\alpha}$	$F_{2AE\alpha}$
	100		0.051	0.050
	200		0.053	0.051
	500		0.049	0.049
Panel B. Size of the tests under two temporary shifts				
	τ_1	τ_2	$t_{2E\alpha}$	$F_{2AE\alpha}$
T = 100				
$\gamma_1 = \gamma_2 = 0.001$	0.3	0.7	0.053	0.051
$\gamma_1 = \gamma_2 = 0.01$	0.3	0.7	0.055	0.053
T = 200				
$\gamma_1 = \gamma_2 = 0.001$	0.3	0.7	0.054	0.052
$\gamma_1 = \gamma_2 = 0.01$	0.3	0.7	0.057	0.052
T = 500				
$\gamma_1 = \gamma_2 = 0.001$	0.3	0.7	0.050	0.051
$\gamma_1 = \gamma_2 = 0.01$	0.3	0.7	0.052	0.052

Empirical sizes are evaluated with 2000 replications. Nominal size is 5%. As in Harvey and Mills (2002), we set $\gamma_1 = \gamma_2$, $\tau_1 = 0.3$, $\tau_1 = 0.7$, $\alpha_1 = 1$, and $\alpha_2 = \alpha_3 = 10$.

$$\varepsilon_t \sim iidN(0, 1) \tag{28}$$

where we set the transition parameters $\tau_1 = 0.3$,$\tau_1 = 0.7$ and let $\gamma_1 = \gamma_2 = (0.001{,}0.01)$. Also $\alpha_1 = 1, \alpha_2 = \alpha_3 = 10, \theta_1 = (0.1{,}1.0), \theta_2 = 1.0, \rho_1 = -0.3, \rho_2 = (-0.3, -0.7, -1.0)$. These specific parameters have been previously used in similar contexts and cover wider range of structural breaks. For a detailed discussion one may refer to Harvey and Mills (2002) and Omay et al. (2020).

For comparison purposes we also computed the power of existing test statistics. The results are presented in Table 3. In particular, we computed power properties of the $F - t_{NL}$ test of Christopoulos and Leon-Ledesma (2010), the F_{LBAE} test of Omay et al. (2018) as well as the $t_{E\alpha}$ and $F_{AE\alpha}$ tests proposed by Omay et al. (2020). These tests also allow for simultaneous changes in the deterministic components of the series along with nonlinear adjustment. Specifically, both Christopoulos and Leon-Ledesma (2010) and Omay et al. (2018) used Fourier forms to model the structural changes. On the other hand, Omay et al. (2020) use a single exponential smooth transition function to model gradual breaks. Christopoulos and Leon-Ledesma (2010) model adjustment towards equilibrium using symmetric ESTAR model. However, Omay et al. (2018) and Omay et al. (2020) use AESTAR model instead.

As can be seen in Table 3, the $t_{2E\alpha}$ and $F_{2AE\alpha}$ tests proposed in this paper outperform other tests in terms of power properties. While the $t_{E\alpha}$ and $F_{AE\alpha}$ tests of Omay et al (2020) as well as the F_{LBAE} test of Omay et al. (2018) have reasonable power for $\gamma_1 = \gamma_2 = 0.001$, the power of both tests fall drastically when $\gamma_1 = \gamma_2 = 0.01$. This is an expected result as lower values of the speed of transition (γ_i) resembles a linear

Table 3 Power analysis of alternative tests

			Existing tests				Newly proposed tests	
$\gamma_1 = \gamma_2$	θ_1	ρ_2	$F - t_{NL}$	F_{LBAE}	$t_{E\alpha}$	$F_{AE\alpha}$	$t_{2E\alpha}$	$F_{2AE\alpha}$
0.001	0.1	-0.3	0.141	0.898	0.272	0.767	0.282	1.000
0.01	0.1	-0.3	0.010	0.010	0.020	0.020	0.215	1.000
0.001	0.1	-0.7	0.101	0.777	0.434	0.505	0.612	0.959
0.01	0.1	-0.7	0.010	0.010	0.010	0.010	0.434	0.949
0.001	0.1	-1.0	0.141	0.757	0.626	0.343	0.674	0.929
0.01	0.1	-1.0	0.010	0.010	0.010	0.010	0.595	0.838
0.001	1.0	-0.3	0.010	0.616	0.454	0.515	0.478	0.989
0.01	1.0	-0.3	0.010	0.010	0.010	0.010	0.444	0.959
0.001	1.0	-0.7	0.089	0.201	0.063	0.161	0.222	0.909
0.01	1.0	-0.7	0.010	0.010	0.010	0.010	0.107	0.818
0.001	1.0	-1.0	0.141	0.191	0.060	0.548	0.565	0.908
0.01	1.0	-1.0	0.010	0.010	0.010	0.010	0.426	0.897

Power properties of the tests are evaluated with 2000 replications.

trend. In fact, it is now well documented that existing tests have relatively reasonable power only for rather slow gradual changes whereas power of these tests drop drastically for moderate and fast breaks. See also discussion in Omay et al. (2020). On the other hand, the $t_{2E\alpha}$ and $F_{2AE\alpha}$ tests preserve reasonable power when $\gamma_1 = \gamma_2 = 0.01$.

4 Data and empirical results

We test the empirical validity of the RIRP hypothesis for 52 countries using monthly data on short-term nominal interest rates and consumer price indices (CPI) that are collected from the OECD and International Monetary Fund's International Financial Statistics (IFS) database over the period 1986:01–2020:12.[5] We have used treasury-bill rates for 12 countries (namely, Austria, Brazil, Egypt, Hong-Kong, Iceland, Italy, Mexico, Pakistan, Philippines, South Africa, Thailand, US), interbank rates for 27 countries (Belgium, Canada, Colombia, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Indonesia, Ireland, Israel, Japan, Korea, Latvia, Lithuania, Luxembourg, Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom), money market rate for 8 countries (namely, Argentina, Bulgaria, Czech Republic, Malaysia, India, Norway, Peru, Singapore) and deposit rate for 6 countries (namely, Bolivia, China, Romania, Russia, Turkey, Uruguay). Treasury-bill rates, money market rates, deposit rates and CPI are collected from the IFS and interbank rates are collected from the OECD. In order to

[5] The start date of the sample period differs from one country to another based on the availability of data for some countries. Also, for some countries, early periods of sample were skipped due to large erratic behaviour of real interest rates.

calculate real interest rates, following Ferreira and León-Ledesma (2007), firstly, we have calculated the quarterly average of the 12- month inflation ahead of period t. Then, we have transformed the annualized monthly interest rate into a compounded quarterly rate, and subtracted inflation from interest rate calculated in that way. We have then computed the RID for 52 countries against the US.

To test the stationarity of the RID series, in addition to the test proposed in this paper, we also applied the conventional ADF test, Kapetanios et al. (2003) (henceforth KSS) test that allow for gradual nonlinear adjustment towards equilibrium, and Omay et al. (2020) (OSH) test that allow for a single temporary structural break in the deterministic component with asymmetric speed of adjustment towards equilibrium. All tests' regressions include only a constant but no trend.[6] Results of these tests are reported in Table 4 below.

Before discussing the results of the unit root tests, we must point out that none of the tests has absolute power over all remaining tests as noticed by Hasanov and Telatar (2011). In fact, if the series are not subject to breaks and do not exhibit nonlinear dynamics the conventional ADF test outperforms other tests. The KSS test has good properties if the adjustment is nonlinear but there is no break in the deterministic components of the series. Similarly, the OSH test and the newly proposed test will have good properties if the series are characterized by simultaneous breaks and nonlinear adjustment, whereas both of these tests will suffer power losses if the series are not subject to either breaks and/or nonlinearities. Therefore, one must consider the results of these tests as complementing each other.[7]

As seen from the Table 4, the conventional ADF unit root test rejects the null hypothesis of unit root only in the case of 17 countries. On the other hand, the KSS test that allows for nonlinear adjustment towards equilibrium rejects the null of unit root for 25 countries, implying that the adjustment towards equilibrium is inherently nonlinear. Allowing for a single temporary break and nonlinearity leads to rejection of the null of unit root in 31 series. This result points to importance of taking account of the structural changes in series. The $t_{2E\alpha}$ test proposed in this paper rejects the null hypothesis of unit root in 45 out of 52 countries, providing evidence in favor of the RIRP hypothesis in majority of the sample countries. However, the $F_{2AE\alpha}$ test rejects the null hypothesis of unit root only in 27 out of 52 countries. Recall that while both $F_{2AE\alpha}$ and $t_{2E\alpha}$ tests allow for gradual temporary shifts in the mean of the series, the $F_{2AE\alpha}$ test assumes asymmetric AESTAR-type adjustment towards the equilibrium whereas the $t_{2E\alpha}$ test assumes symmetric ESTAR-type adjustment. Notice also that whenever the $F_{2AE\alpha}$ test rejects the null hypothesis of unit root, the $t_{2E\alpha}$ test also rejects the null hypothesis. Thus, these results suggest

[6] To save space, we do not report estimates of the coefficients of the nonlinear trend function, which are available upon request from the corresponding author. To depict the nature of the gradual shifts in the series under consideration, we plot the graphs of the RID series of all sample countries along with estimated double ESTR trend functions in Fig. 2 in Appendix A. The fitted trend functions capture major swings in the series quite well in most of the countries. Visual inspection of the RID series and comparison with simulated trend series in Fig. 2 reveal the importance of taking into account of structural breaks in analysing RIRP hypothesis for many countries.

[7] See also discussions in Appendix B.

Table 4 Unit root test results for real interest rate parity

	Existing Unit Root Tests			Newly Proposed Tests	
	τ_{μ}	t_{NL}	$t_{E\alpha}$	$t_{2E\alpha}$	$F_{2AE\alpha}$
Argentina	-2.542	**-2.768***	-2.792	-3.415	7.154
Austria	-2.492	-2.119	**-3.823****	**-4.499****	10.640
Belgium	-2.382	-2.477	**-3.726***	**-5.142****	**16.000****
Bolivia	-2.227	-2.534	-2.629	**-4.301***	11.777
Brazil	-2.523	**-4.838*****	**-5.047*****	**-6.207*****	**20.124****
Bulgaria	-2.303	-2.374	**-3.549***	-3.663	7.227
Canada	-1.844	**-2.688***	**-4.352****	**-5.156****	**21.468****
China	**-2.917****	-2.401	-2.938	**-4.292***	9.270
Colombia	**-3.368****	**-3.034****	**-6.782*****	-3.427	7.066
Czech Republic	-2.181	-2.197	-2.888	**-4.114***	9.663
Denmark	-2.365	**-5.236*****	**-6.840*****	**-9.164*****	**41.883*****
Egypt	-2.521	**-3.033****	-1.906	-3.297	8.662
Estonia	**-2.592***	**-2.915***	**-4.780*****	**-4.572****	8.545
Finland	-2.280	**-3.459****	**-5.479*****	**-7.374*****	**27.165*****
France	-2.072	-2.536	**-4.657*****	**-5.104****	**13.683***
Germany	-2.322	-2.456	-3.398	**-4.596****	10.684
Greece	-2.296	-2.488	-3.275	**-4.468***	10.098
Hong-Kong	-2.081	-2.475	-3.419	**-4.246***	9.124
Hungary	**-3.339****	**-3.462****	**-4.816*****	**-4.407***	9.743
Iceland	**-3.197****	**-3.727*****	**-6.357*****	**-6.579*****	**21.954****
India	-1.934	**-5.649*****	**-6.215*****	**-5.761****	**19.820****
Indonesia	**-2.717***	**-3.313****	-3.225	-3.690	7.148
Ireland	**-3.020****	**-8.999*****	**-7.595*****	**-9.064*****	**45.377*****
Israel	**-2.616***	-2.019	-3.106	**-5.412****	**14.746***
Italy	-2.194	-2.255	**-6.331*****	**-6.371*****	**21.599****
Japan	-2.205	**-3.791*****	**-4.806*****	**-3.949***	7.904
Korea	-2.544	-2.456	**-3.629***	**-4.742****	11.342
Latvia	**-2.871***	**-4.700*****	-2.056	**-5.319****	**20.830****
Lithuania	-2.099	**-2.804***	**-5.914*****	**-6.347*****	**20.243****
Luxembourg	-2.467	-2.393	-3.474	**-3.906***	7.698
Malaysia	**-3.421****	**-5.097*****	**-4.840*****	**-5.084****	**13.389***
Mexico	-2.073	-1.981	**-3.768***	**-5.449****	**17.347****
Netherlands	-2.009	-1.456	-3.447	**-3.960***	10.525
Norway	-2.225	-2.033	-3.131	**-5.409****	**15.232***
Pakistan	-2.414	-2.422	-2.339	-2.394	7.041
Peru	**-2.954****	-2.649	**-3.643***	**-6.599*****	**22.336****
Philippines	**-3.030****	-2.484	**-4.685*****	**-6.123*****	**18.961****
Poland	-2.200	-2.135	**-4.299****	**-4.834****	12.078
Portugal	-2.378	-2.619	-3.471	**-5.075****	**15.280****
Romania	-2.462	-2.125	**-3.838****	**-6.646*****	**21.999****
Russia	-2.468	**-2.852***	-3.295	**-4.156***	8.699

Table 4 (continued)

	Existing Unit Root Tests			Newly Proposed Tests	
	τ_{μ}	t_{NL}	$t_{E\alpha}$	$t_{2E\alpha}$	$F_{2AE\alpha}$
Singapore	**-3.271****	**-3.537*****	**-5.107*****	**-4.278***	9.129
Slovakia	-2.497	-1.125	**-7.084*****	**-6.281*****	**21.019****
Slovenia	-2.128	-1.899	-2.643	-3.831	7.834
South Africa	**-2.881****	**-3.267****	**-4.050****	**-4.858****	11.834
Spain	-2.361	**-3.381****	**-3.454***	**-6.352*****	**20.652****
Sweden	**-2.664***	**-7.854*****	**-7.294*****	**-9.252*****	**49.550*****
Switzerland	-2.269	**-2.680***	-3.382	**-4.411****	9.709
Thailand	**-2.765***	**-4.300*****	**-4.610*****	**-5.015****	**12.624***
Turkey	**-3.073****	-2.358	**-3.592***	**-5.692****	**16.308****
United Kingdom	-1.918	-2.072	-2.662	**-3.951***	**14.891***
Uruguay	-2.089	**-5.701*****	**-4.819*****	**-6.961*****	**24.251****

***, ** and * denote the rejection of the unit root null hypothesis at 1%, 5% and 10% significance levels, respectively. In order to be seen more clearly, computed test statistics of the stationary series are marked in bold. τ_{μ}, t_{NL}, $t_{E\alpha}$, $t_{2E\alpha}$ and $F_{2AE\alpha}$ denote the conventional ADF test, KSS test, the OSH test, and the unit root tests with ESTAR- and AESTAR-type adjustments proposed in this study, respectively

that the adjustment of deviations towards the equilibrium can be best described by an ESTAR-type nonlinearity rather than the asymmetric AESTAR-type nonlinearity. This finding suggests that the adjustment of the domestic real interest rates towards global rates depend on the size but not on the sign of the deviations from the global rates. Notice also that the newly developed $t_{2E\alpha}$ test rejects the null hypothesis of unit root for 10 countries (namely, Bolivia, Czech Republic, Germany, Greece, Hong-Kong, Luxembourg, Netherlands, Norway, Portugal, and the United Kingdom) for which no other unit root test procedure employed in this study could reject the null hypothesis of unit root. All in all, our findings reveal superiority of the proposed unit root test procedures over the existing tests.

The results of these tests have several important implications. First, allowing for nonlinearity results in more frequent rejection of the unit root, suggesting that interaction between real interest rates among countries are in fact nonlinear. Second, allowing for breaks along with nonlinear adjustment produced more support for the RIRP hypothesis, implying that shocks have caused to significant breaks in the RIDs although these breaks might have been temporary. Third, our results imply that the speed of adjustment of the domestic interest rates towards the global rate depends on the size but not the sign of the deviation from the global rates. This suggests that big deviations are corrected faster than the small deviations whereas there is no difference in the speed of adjustment of positive versus negative deviations. Fourth, we find that the RIRP hypothesis holds for majority of the countries during the analysed period. In fact, we were not able to reject the null of unit root only in two cases, namely in the cases of Pakistan and Slovenia. All in all, our findings imply that the goods and financial markets of the majority of the sample countries are well integrated to the world markets.

5 Conclusion

This study investigates the empirical validity of the RIRP hypothesis for 52 countries by testing stationarity of RIDs. As known, RIDs might be subject to both nonlinear adjustments and structural changes and thus might be subject to considerable short-run deviations from the equilibrium levels. Many authors have proposed unit root test procedures that allow for simultaneous permanent structural breaks and nonlinear adjustments for testing stationarity of time series. However, permanent structural breaks is not compatible with the RIRP hypothesis, as the RIRP hypothesis assumes that capital flows and free arbitrage activities tend to equalize real interest rates across countries. Taking account of the fact that both asymmetric adjustment and temporary shifts may better characterize dynamics of RIDs, we propose a new test that takes into account two temporary gradual shifts together with asymmetric adjustment towards equilibrium which is built on the recently developed OSH test.

For comparison purposes, we also applied the conventional ADF test, the KSS test that allows for gradual nonlinear adjustment towards equilibrium, and recently developed OSH test that allows for temporary smooth structural change in the deterministic component with asymmetric speed of adjustment towards equilibrium. Test results indicate that allowing for temporary structural changes and nonlinearities in unit root tests results in more frequent rejection of the null hypothesis of unit root, consistent with the RIRP hypothesis. In particular, our newly developed unit root test procedure highly outperforms ADF, KSS and OSH tests in terms of number of stationary RIDs. Moreover, the new test procedure reject the null hypothesis of unit root for 10 countries for which no other unit root tests employed in this study can reject the null hypothesis of unit root. These results imply that RIDs of many countries vis-a-vis US might be inherently nonlinear with transitory shifts in the equilibrium level. Overall, our unit root test results indicate that RIRP hypothesis is supported for 50 out of 52 countries in the sample.

Some important points emerge. First, our results imply that while shocks may cause to significant shifts in the real interest differentials in the short run, such shifts are temporary in nature and the RIDs of most countries will come back to their pre-shock levels in the long run. Second, our results imply that the adjustment of deviations towards equilibrium might be inherently nonlinear but symmetric. Specifically, our findings imply that the speed of adjustment depends only on the size of the disequilibrium but not on the sign. This suggests that large deviations in RIDs tend to provoke equilibrating responses whereas adjustments tend to occur faster as the shocks are bigger. On the other hand, our results point to no significant difference in the speed of adjustment of negative versus positive deviations. Finally, our results support the view that most emerging economies are financially and economically integrated to the world economy, implying that policies aimed at affecting real economic variables via interest rates will have only short-lived effects.

Appendix A. Graphs of the RID series and estimated trend functions

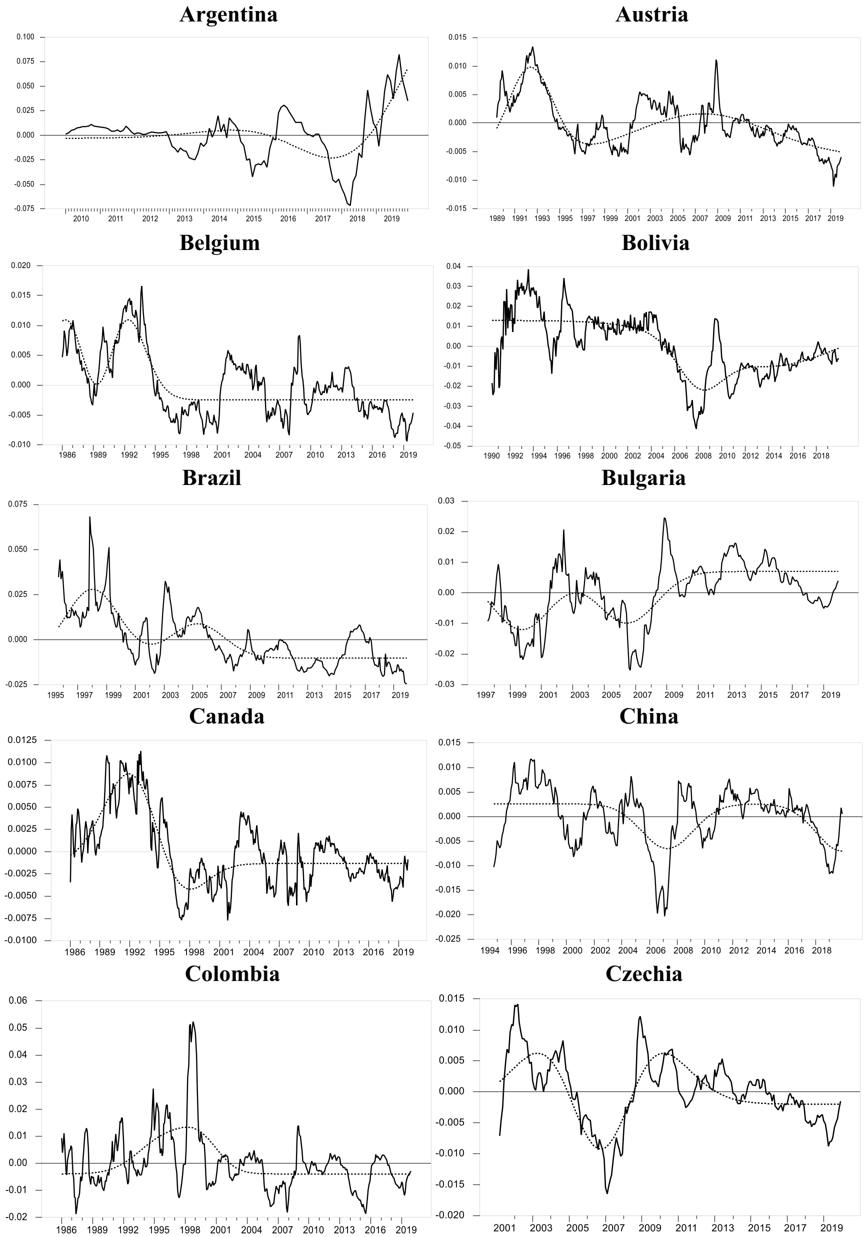


Fig. 2 RIDs (solid line) and estimated double ESTR trend functions (dashed line)

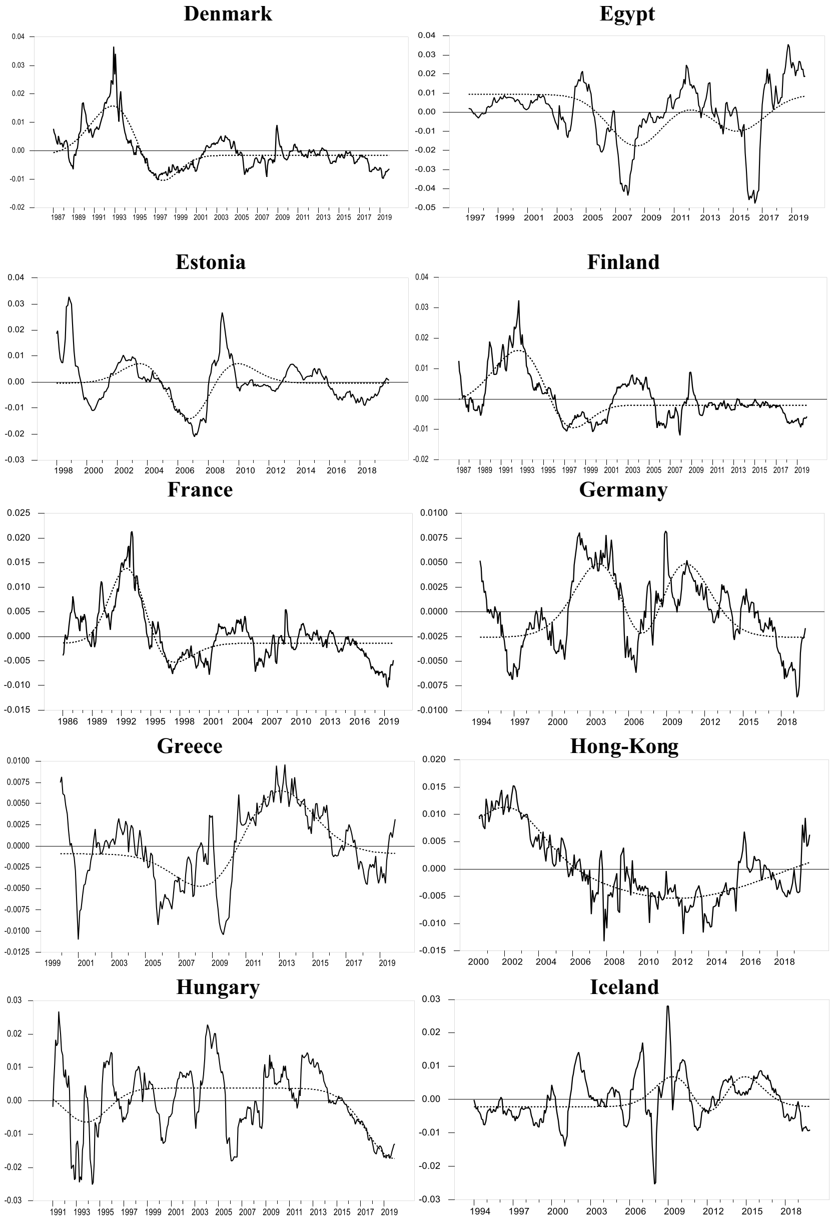


Fig. 2 (continued)

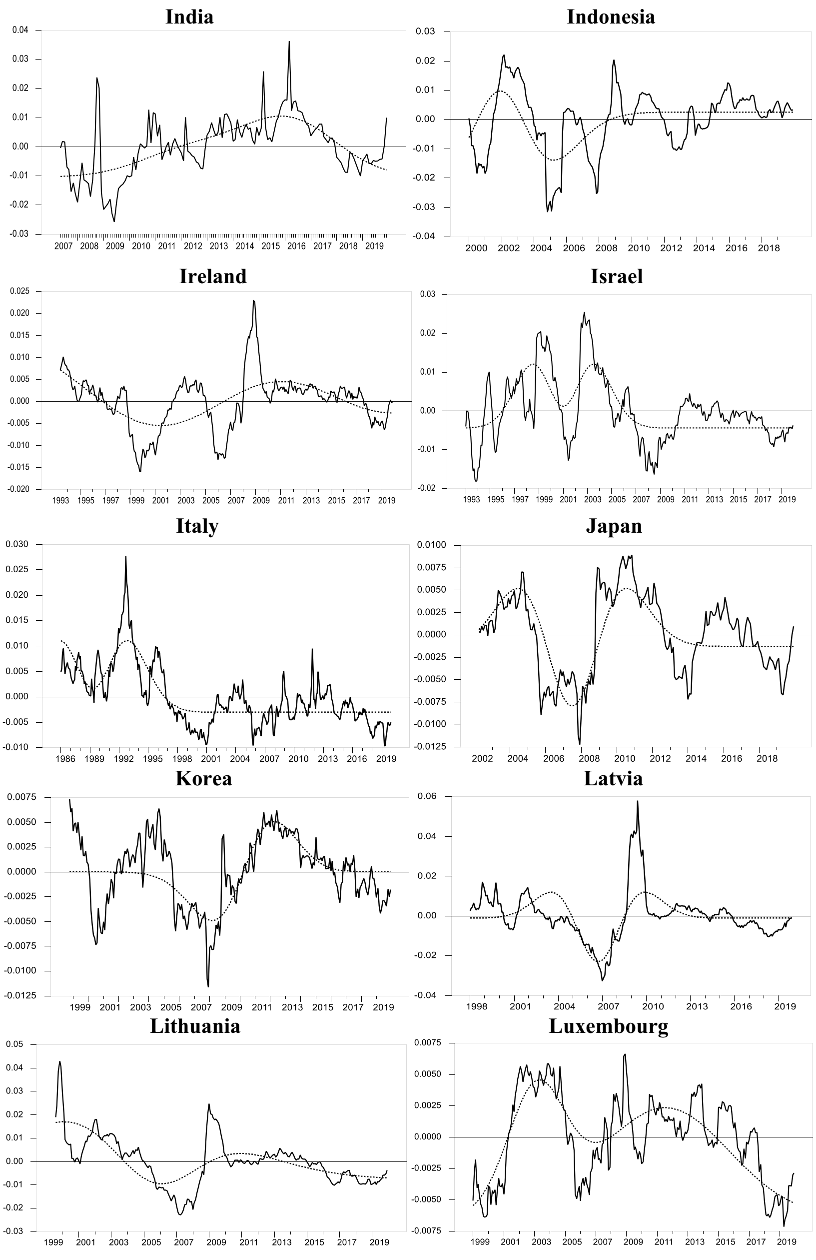


Fig. 2 (continued)

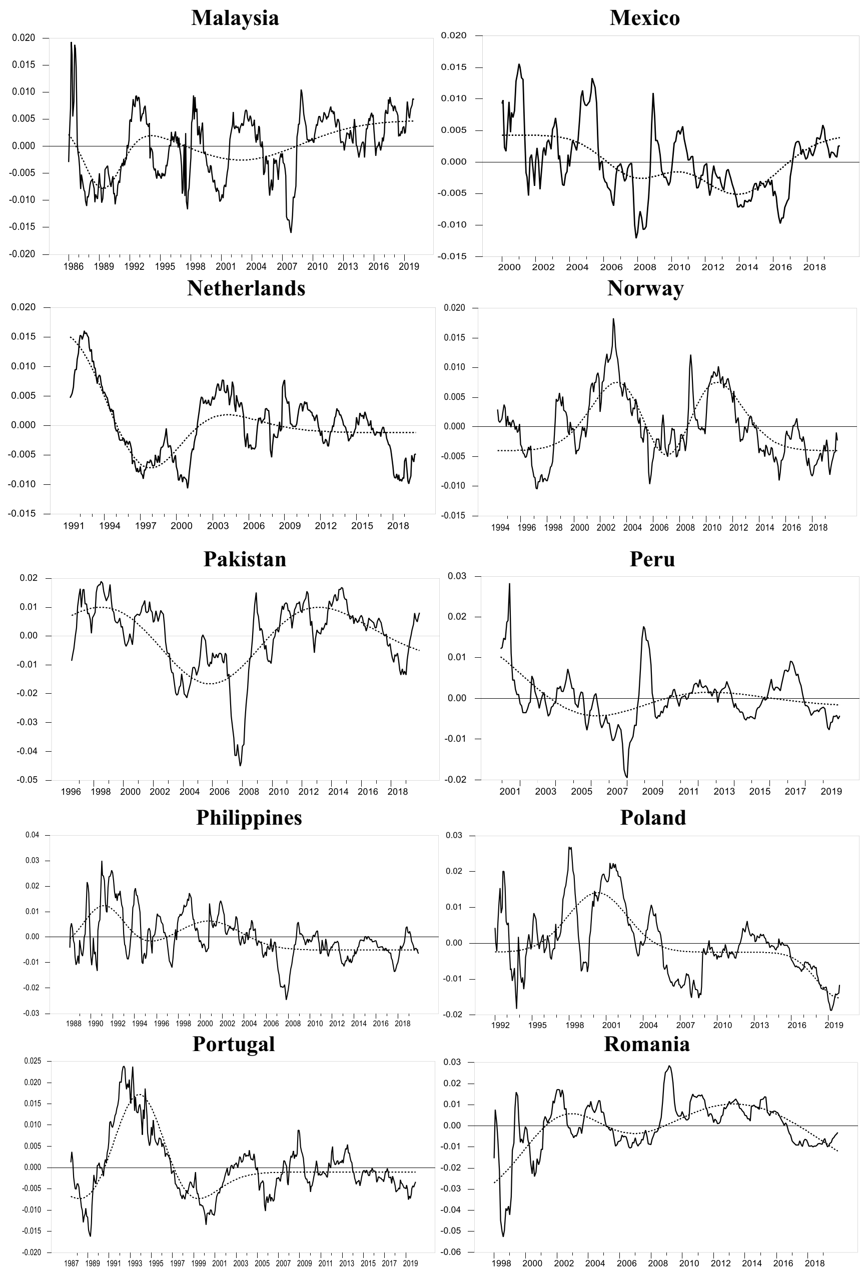


Fig. 2 (continued)

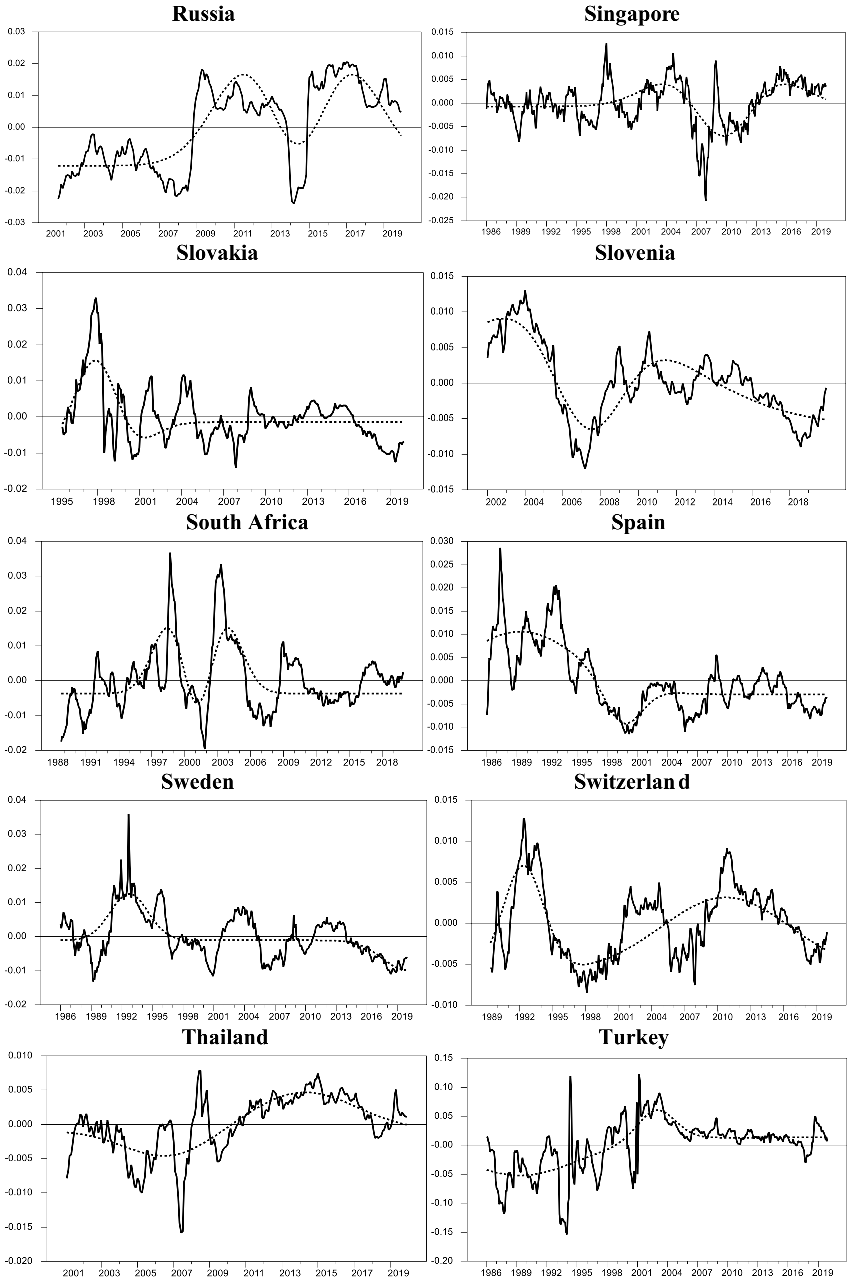


Fig. 2 (continued)

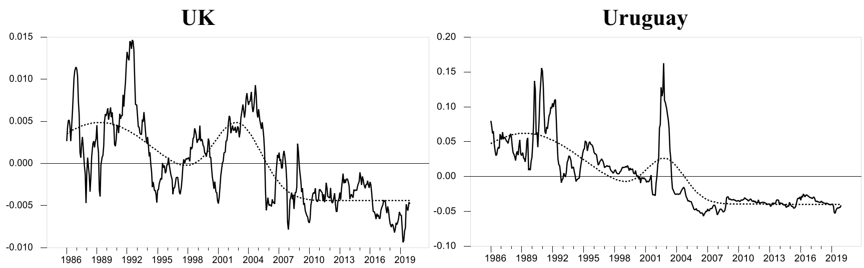


Fig. 2 (continued)

Appendix B. Testing gradual change in the deterministic component of the series

In practice, researchers might be concerned about choosing the appropriate test for the time series under consideration: should a test allowing for a break be applied or conventional tests are just right? If there is a break, should it be a single break or a multiple break test? The issue becomes more complicated when breaks are modelled using nonlinear trend functions that allow for continuous rather than abrupt changes in the trend of the series.

One practical solution is to apply alternative tests sequentially; first apply any conventional unit root test (e.g., the ADF), rejection of which would be seen as evidence of stationarity. However, if these tests do not reject the null, one would apply a test which allows for a single transitory break in the series (e.g., Omay et al. 2020). If this test does not reject the null of unit root as well, one may proceed to the tests proposed in this paper. However, as shown by Nunes et al. (1997), this approach places the null hypothesis in considerable jeopardy: the overall size of this multi-stage test can exceed the nominal size considerably.[8] Additionally, this approach adds considerable estimation burden on practitioners, mainly stemming from sequential estimation of nonlinear models. In fact, in the second step the researcher needs to estimate the nonlinear trend function using a single transition function. If the null hypothesis is not rejected again, the researcher would re-estimate the nonlinear trend function with two transition functions and repeat the test. Or before estimating the trend function with two transition functions, one would ideally test the adequacy of the estimated trend function against remaining non-linearities in the deterministic component of the series (see, for example, Eitrheim and Teräsvirta 1996). Alternatively, one may alleviate estimation burden by pre-testing the functional form of the trend function following Lin and Teräsvirta (1994), and depending on the results proceed with the selected trend function. As this procedure does not entail estimating a nonlinear model, we suggest using this procedure rather than applying the tests sequentially.

Consider the following DGP:

[8] We would like to thank an anonymous reviewer for reminding us possible size distortions associated with such a sequential testing approach.

$$y_t = \phi(t) + u_t \tag{A.1}$$

where the deterministic $\phi(t)$ trend function may have a single transitory shift

$$\phi(t) = \alpha_1 + \alpha_2 S_1(\gamma_1, \tau_1) \tag{A.2}$$

or two shifts:

$$\phi(t) = \alpha_1 + \alpha_2 S_1(\gamma_1, \tau_1) + \alpha_3 S_2(\gamma_2, \tau_2) \tag{A.3}$$

where the transition functions are as defined in the text. One may test linearity of the deterministic component by replacing the ESTR functions with their proper linear approximations (see, for example, Lin and Teräsvirta 1994). For example, replacing the ESTAR function S_1 with its first-order Taylor series approximation one obtains the following auxiliary regression:

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + e_t \tag{A.4}$$

>where e_t comprises the deviations u_t in eq. (A.1) as well as the error term arising from the Taylor approximation. One may test linearity by testing the null hypothesis $H_0 : \beta_1 = \beta_2 = 0$, rejection of which would provide an evidence in favour of a single transitory shift. However, if there are two additive ESTR transition functions as given in eq. (A.3) one may use higher order approximations. Consider, for example\ the following regression equation:

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4 + e_t \tag{A.5}$$

One may test linearity of the deterministic component by testing the null hypothesis:

$$H_{10} : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$

Rejection of which would provide an evidence of nonlinearity in the trend of the series. If H_{10} is rejected, one may proceed to test whether double transition functions are necessary or a single transition function is adequate to capture the nonlinearities in the trend. In particular, if H_{10} is rejected, one may test:

$$H_{20} : \beta_3 = \beta_4 = 0$$

$$H_{30} : \beta_1 = \beta_2 = 0 \middle| \beta_3 = \beta_4 = 0$$

Rejection of H_{20} would imply that a single transition function might not be adequate for capturing the nonlinearities in the deterministic component of the series. If H_{20} is not rejected but H_{30} is rejected, single transition might be more appropriate to model the deterministic component. In fact, notice that the null H_{30} tests linearity of the trend function against the single ESTR-type gradual shift. Therefore, rejection of both H_{10} and H_{30} along with no-rejection of H_{20} would imply that a single ESTR-type nonlinearity is sufficient to capture nonlinearities in the trend function. Notice also that if the null H_{10} is not rejected, one may still proceed to test H_{30} using the regression eq. (A.4), rejection of which would also suggest a single ESTR-type nonlinearity in the trend. However, rejection of both H_{30} and H_{20} would suggest that single ESTR type nonlinearity might be inadequate to capture nonlinearities in the trend of the series.[9]

Table 5 below reports the results of these hypotheses tests for the RID series.

Table 5 Break test results

Countries	H_{10}	H_{20}	H_{30}
Argentina	15.555 [0.000]	17.009 [0.000]	11.071 [0.000]
Austria	82.645 [0.000]	58.735 [0.000]	80.894 [0.000]
Belgium	68.613 [0.000]	29.026 [0.000]	95.046 [0.000]
Bolivia	99.311 [0.000]	63.056 [0.000]	100.152 [0.000]
Brazil	73.058 [0.000]	1.790 [0.169]	143.541 [0.000]
Bulgaria	27.593 [0.000]	16.195 [0.000]	34.992 [0.000]
Canada	75.334 [0.000]	45.253 [0.000]	86.509 [0.000]
China	28.845 [0.000]	51.134 [0.000]	4.915 [0.008]
Colombia	19.080 [0.000]	14.056 [0.000]	22.644 [0.000]
Czech Republic	21.617 [0.000]	28.471 [0.000]	11.845 [0.000]
Denmark	54.553 [0.000]	39.331 [0.000]	58.385 [0.000]
Egypt	11.178 [0.000]	5.660 [0.004]	16.145 [0.000]
Estonia	17.152 [0.000]	21.564 [0.000]	11.005 [0.000]
Finland	50.661 [0.000]	32.942 [0.000]	58.818 [0.000]
France	83.226 [0.000]	68.391 [0.000]	73.575 [0.000]
Germany	36.475 [0.000]	11.891 [0.000]	56.989 [0.000]
Greece	25.194 [0.000]	43.019 [0.000]	5.446 [0.005]
Hong-Kong	169.001 [0.000]	7.211 [0.000]	313.909 [0.000]
Hungary	24.549 [0.000]	20.528 [0.000]	25.663 [0.000]
Iceland	14.711 [0.000]	6.486 [0.002]	22.149 [0.000]
India	28.906 [0.000]	13.083 [0.000]	38.487 [0.000]
Indonesia	5.834 [0.000]	4.117 [0.017]	7.358 [0.000]
Ireland	34.586 [0.000]	67.707 [0.000]	1.029 [0.358]
Israel	34.925 [0.000]	43.946 [0.000]	20.436 [0.000]
Italy	112.491 [0.000]	66.227 [0.000]	120.076 [0.000]
Japan	9.673 [0.000]	16.196 [0.000]	2.751 [0.066]
Korea	13.255 [0.000]	26.402 [0.000]	0.089 [0.914]
Latvia	8.549 [0.000]	14.761 [0.000]	2.114 [0.123]
Lithuania	39.633 [0.000]	36.459 [0.000]	33.105 [0.000]
Luxembourg	49.364 [0.000]	34.536 [0.000]	50.570 [0.000]
Malaysia	16.348 [0.000]	0.802 [0.449]	31.925 [0.000]

[9] Here we must note that the procedures outlined here is not meant to propose a procedure to test for and chose the most appropriate model for the deterministic component of the series, for which researchers may refer to the relevant literature. See, e.g., Lin and Teräsvirta (1994), Perron et al. (2017).

Table 5 (continued)

Countries	H_{10}	H_{20}	H_{30}
Mexico	26.643 [0.000]	6.715 [0.001]	44.429 [0.000]
Netherlands	108.798 [0.000]	176.234 [0.000]	20.458 [0.000]
Norway	22.005 [0.000]	3.786 [0.024]	39.504 [0.000]
Pakistan	41.854 [0.000]	58.696 [0.000]	17.694 [0.000]
Peru	19.199 [0.000]	31.407 [0.000]	5.509 [0.005]
Philippines	35.053 [0.000]	18.844 [0.000]	46.860 [0.000]
Poland	38.616 [0.000]	17.015 [0.000]	54.932 [0.000]
Portugal	58.423 [0.000]	95.545 [0.000]	14.321 [0.000]
Romania	37.401 [0.000]	7.783 [0.000]	63.708 [0.000]
Russia	45.383 [0.000]	1.682 [0.188]	88.529 [0.000]
Singapore	13.897 [0.000]	13.949 [0.000]	13.013 [0.000]
Slovakia	31.171 [0.000]	26.505 [0.000]	30.492 [0.000]
Slovenia	47.002 [0.000]	39.950 [0.000]	39.578 [0.000]
South Africa	20.581 [0.000]	23.727 [0.000]	15.536 [0.000]
Spain	152.066 [0.000]	72.356 [0.000]	171.384 [0.000]
Sweden	36.214 [0.000]	24.916 [0.000]	42.493 [0.000]
Switzerland	30.430 [0.000]	59.909 [0.000]	0.719 [0.487]
Thailand	52.043 [0.000]	43.146 [0.000]	44.277 [0.000]
Turkey	85.476 [0.000]	35.965 [0.000]	115.112 [0.000]
United Kingdom	82.978 [0.000]	0.683 [0.505]	165.532 [0.000]
Uruguay	163.146 [0.000]	14.050 [0.000]	293.338 [0.000]

Figures in brackets are *p*-values

List of abbreviations ADF: Augmented Dickey-Fuller test; AESTAR: Asymmetric exponential smooth transition autoregressive; CPI: Consumer price indices; DGP: Data generating process; ESTAR: Exponential smooth transition autoregressive; ESTR: Exponential smooth transition; IFS: International Financial Statistics; KSS: Kapetanios et al. (2003) nonlinear unit root test; OSH: Omay et al. (2020) nonlinear unit root test under structural break; PPP: Purchasing power parity; RID: Real interest rate differentials; RIRP: Real interest rate parity; UIP: Uncovered interest parity

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Data availability All data analysed during this study are included in this published article [and its supplementary information files]. Test procedures developed in this paper are available as apps at: https://tolgaomay.shinyapps.io/DESTR_Test_Model_A_AESTAR/,https://tolgaomay.shinyapps.io/DESTR_Test_Model_B_AESTAR/, https://tolgaomay.shinyapps.io/DESTR_Test_Model_C_AESTAR/, https://tolgaomay.shinyapps.io/DESTR_Test_Model_A/, https://tolgaomay.shinyapps.io/DESTR_Test_Model_B/, https://tolgaomay.shinyapps.io/DESTR_Test_Model_C/.

Declarations

Competing interests The authors declare no competing interests.

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