

Re-examining crude oil and natural gas price relationship: Evidence from time-varying regime-switching models

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ABSTRACT

In this paper, we re-examine the relationship between crude oil and natural gas prices. Using a more flexible modelling approach, we find that the relationship between these two prices are more complex than previously documented. Specifically, we find that both the short-run and long-run relationships are highly nonlinear and shifted considerably over time. The effects of oil prices on gas prices fell significantly and became less volatile after 2011. We also find a significant shift and nonlinearity in the short-run response of gas prices to demand and supply conditions. Overall, our results imply that the natural gas prices did not decouple from crude oil prices in the sense that the long-run equilibrium relationship between the two prices did not break completely, but instead moved to a new level.

1. Introduction

Natural gas prices have usually attracted great attention from both economists and policy authorities, especially during periods when gas prices surge rapidly. Higher gas prices entail significant social costs as households depend mainly on natural gas for residential heating in most advanced countries. Surge in gas prices may affect households indirectly through rising electricity bills as natural gas is the most important fuel for power generation. Higher natural gas prices are also detrimental for overall economic activity as natural gas is the second largest source of energy after crude oil for developed countries.¹ Moreover, natural gas is seen as the key transition fuel to renewable energy sources and net zero economy (see also discussions in [Szafranek and Rubaszek, 2023](#)). Given its importance for modern economies, gas price dynamics have been widely examined in the literature. Yet, natural gas pricing remains to be one of the controversial issues in energy economics (e.g., [Stern, 2012a](#); [Stern and Rogers, 2013](#)). Natural gas prices have usually been linked to oil products and/or crude oil prior to 2010s.² Linkage to oil prices, commonly referred to as oil-indexation, was usually justified by the

arguments that oil products and natural gas are close substitutes and users can easily switch from one type of fuel to other. However, after the 2008–2009 global financial crisis, hub-based gas prices diverged significantly from the oil-indexed prices, which in turn lead to heated debates on whether gas prices decoupled from oil prices or not (see, for example, [Erdős, 2012](#); [Brigida, 2014](#); [Batten et al., 2017](#); [Zhang and Ji, 2018](#)).

The knowledge of the true nature of the link between oil and gas prices is central to many practical decision-making processes. For example, long-term gas contracts use some price formation mechanisms that are usually based on oil prices or hub prices or a hybrid pricing mechanism using both hub and oil prices. The empirical relationship between oil and hub-based gas prices could be used in determining optimal weights of hub and oil prices in price setting rules under such long-term contracts.³ If there exists a stable long-run relationship between crude oil and hub-based gas prices, oil indexation may reduce risks associated with excessive price fluctuations as the global oil markets are deeper and more liquid when compared to regional gas markets. Also, in this case both consumers and producers may use the observed oil

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¹ According to [BP Statistical Review of World Energy \(2022\)](#), natural gas accounted around 24.4% of the global primary energy consumption in 2021. Natural gas accounted for 32.7% of primary energy consumption in North America and about 25% in Europe.

² See [Stern \(2012b\)](#) for a thorough historical survey of gas pricing across different regions.

³ In fact, the surge in European gas prices during 2022 raised discomforts with high gas prices and lead to increasing demands for a search of new benchmark for European gas prices. See, for example, [European Commission \(2022\)](#).

and gas price relationship to hedge against adverse price changes. Similarly, traders in commodity markets may develop profitable trading strategies using the empirical oil and gas price relationships. Price relationship is also central to decisions on optimal fuel mix by energy importing countries and big energy consumers. In fact, cost minimization and optimization behaviour of economic agents depend on relative prices. Relative fuel prices are also crucial for resource allocation decisions of hydrocarbon producers. Oil and gas price relationship might also be used by policymakers in assessing energy policies as well as when imposing taxes both on production and consumption (see also discussions in Yücel and Guo, 1994; Ramberg and Parsons, 2012; Brigida, 2014; Caporin and Fontini, 2017).

The existence of an inherent long-run equilibrium relationship between oil and gas prices, however, has been questioned in the face of sporadic, but sometimes persistent divergences of prices from levels implied by substitutability, and it has been argued that gas prices might have been decoupled from oil prices (e.g., Stern and Rogers, 2013). As pointed out by Ramberg and Parsons (2012), “decoupling”, as used in the energy literature, may have three alternative interpretations. The first interpretation is that the relationship has broken only temporarily and will return to the previous level with time lapse. For instance, Asche et al. (2017) find that gas prices decouple from oil prices with seasonal demand whereas the industry’s 10–1 rule-of-thumb holds during usual off-peak periods. The second interpretation of decoupling is that the old relationship has broken and a new relationship is established. Chevallier and Ielpo (2013), Wakamatsu and Aruga (2013), Zhang and Ji (2018), among others, find time-varying relationship between gas and oil prices. The third interpretation is that the relationship has been broken permanently and oil and gas prices will never recouple again (e.g., Stern, 2014).

These alternative interpretations of decoupling have quite different implications for both practitioners and policy authorities. In fact, there is no need to alter the existing energy mix if breakdown of the price relationship is temporary. In this case, the observed relationship between oil and gas prices could be used in determining optimal weights of hub and oil prices in price setting rules under long-term contracts. The temporary nature of breakdown also points to the existence of profitable trading strategies. As the prices tend to revert to the previous equilibrium relationship, traders can correctly conjecture both the direction and magnitude of price changes. On the other hand, if the prices have decoupled permanently, oil prices will become completely irrelevant for producers and consumers of natural gas. As the existence of the link between the crude oil and gas prices was explained by substitutability of oil products and gas, the disappearance of the link would imply that the possibility of substitution has been eliminated and even price incentives will not lead to fuel switch. Stern and Rogers (2013) argued that a combination of some important developments, including efficiency gains brought by emergence of modern gas-burning equipment as well as tightening environmental standards regarding pollutant emissions undermined the rationale for the linking gas prices to oil prices. It then follows that the same developments indeed weakened the substitutability of the fuels, which have important implications regarding analysis of energy consumption dynamics as well as design of optimal energy policies. If the oil and gas price relationship changes over time, consumers will need to adjust optimal energy mix taking account of the new equilibrium relationship. Changing price relationship will also affect investment decisions of producers as well as alter trading strategies of traders. Also, if the equilibrium oil and gas price relationship is subject to shifts, both the producers and consumers need to adopt flexible production and consumption strategies that allow them costless or low-cost adjustment towards the new equilibrium levels. For example, investments in multi-fuel technology might be more cost-effective in the long run once one acknowledges the possibility of a significant change in relative fuel prices.

Another related issue is whether the relationship between the prices is symmetric or nonlinear. Indeed, adjustment costs, imperfect

competition and other rigidities may cause to nonlinearities in price responses. For example, if the price differences are not big enough to trigger fuel switch, consumers will not alter their fuel consumptions, and hence, prices will not move to correct the observed discrepancies. In this case, the relationship between the prices will depend on the size of deviations from the equilibrium level. Similarly, strategic firm behaviour, inventory levels, production lags and market structure may trigger the so-called “rockets and feathers” effect, i.e., faster price adjustments to oil price increases than to oil price decreases (see discussions in Atil et al., 2014, and references therein). Brigida (2014), Asche et al. (2017), Batten et al. (2017), among others, found that the oil-gas price relationship exhibits significant nonlinearities.

In this paper, we analyse the relationship between the crude oil and natural gas prices by allowing for possibility of simultaneous structural breaks and nonlinearities. While both of these features were investigated in the literature, to the best of our knowledge, time-varying nonlinearities are not yet analysed. Specifically, researchers who studied structural changes (e.g., Caporin and Fontini, 2017; Zhang and Ji, 2018; Szafranek and Rubaszek, 2023) ignored possible nonlinearities whereas those who analysed nonlinearities (e.g., Brigida, 2014; Geng et al., 2016; Asche et al., 2017) ignored shift in the relationship over time. If the relationship between the variables is characterized by simultaneous breaks and nonlinearities, failing to account for one of these features would lead to misspecification of the other feature as well (e.g., the failure to model nonlinearity will lead to misspecification of the breaks and vice-versa). In addition, it is well established that specification tests produce misleading results suggesting nonlinearity (structural break) when the data is actually generated by a structural break (nonlinear) model (see, for example, van Dijk, 1999; Carrasco, 2002). Another contribution of the paper is that we analyse structural breaks and nonlinearities both in the long- and short-run relationships between the crude oil and natural gas prices. The existing literature has focused on breaks or nonlinearities only in the long-run relationship between the prices but failed to allow for time- or regime-varying short-run relationships among the variables under investigation. For this purpose, we use time varying smooth transition regression models (TV-STR) proposed by Lundbergh et al. (2003) to model simultaneous structural changes and possible nonlinearities in the oil and gas price relationships.

The TV-STR modelling approach used in this paper has several advantages over competing nonlinear models (see also discussions in Hasanov et al., 2010). First, the TV-STR modelling approach allows researchers to test whether a time-varying or a nonlinear model is sufficient to describe the relationship between the variables or more complex models that allow for time-varying nonlinearity are needed to analyse the relationship. The TV-STR models nests linear, nonlinear, structural change, and time-varying nonlinear models as special cases. The model specification tests allow researchers to select the most appropriate structure for the data under investigation. Alternative nonlinear models such as the threshold and Markov regime switching models, on the other hand, allow either a nonlinearity or a structural change but not time-varying nonlinearity in the dynamics of variables. Unlike these alternative nonlinear models, the TV-STR modelling approach also allows testing statistical significance of structural breaks and nonlinearities both before specification and after estimation of the model. Second, unlike threshold and Markov regime switching models that impose a pre-specified nonlinear structure, the STR modelling approach allows to choose the most appropriate structure among alternative specifications. In particular, specification tests allow one to decide whether a logistic or an exponential transition function is more appropriate to model both the structural breaks and nonlinearities. As further elaborated below, these two transition functions capture quite different and wide range of dynamics compatible with the existing literature. For example, the exponential transition function governing structural change imposes a temporary structural break whereas a logistic function imposes a permanent break. Similarly, an exponential transition function governing nonlinearities in the relationship is more appropriate when small and

big price changes have different effects, which is justified by the adjustment costs or market rigidities. On the other hand, the “rockets and feathers” effect suggests that positive and negative price changes have asymmetric impacts, which is more aptly modelled by a logistic transition function. Third, TV-STR models allow for gradual regime shifts and structural changes, which are theoretically more appealing than simple threshold models which impose abrupt changes. In addition, the TV-STR models nests these abrupt change models as well. Considering these advantages, we employ TV-STR modelling approach to investigate oil gas price relationship. Using model specification tests, we find that the both the long-run and short-run relationships between WTI and Henry Hub prices are best modelled by simultaneous structural break and regime-wise nonlinearities. Our results suggest that while the Henry Hub prices moved, on average, one-to-one with WTI prices in the long run before 2005, the effect of oil prices on gas prices has halved after 2011. We also find significant time-varying nonlinearities in the short-run response of gas prices to oil prices as well as to supply and demand conditions.

The rest of the paper is organized as follows. In the next Section 2 we review the existing literature on the relationship between crude oil and natural gas prices. The econometric model and its specification procedure is outlined in the Section 3. Model specification tests and estimation results are presented in the Section 4. Section 5 discusses the results, and then Section 6 concludes.

2. Literature review

Oil and gas price relationship is debatable both on theoretical and empirical grounds. According to the thermal parity approach, as both oil and natural gas are energy carriers, relative prices must roughly reflect their respective energy contents. For example, as a barrel of oil contains approximately 5.8 million BTU, the price of a barrel of oil should roughly equal six times the price of a million BTU of natural gas (Brown and Yücel, 2008).⁴ However, most economists and market participants acknowledge that oil and gas price relationship is not determined solely by the energy content equivalence as these fuels have different costs of production, transportation, storage as well as different environmental costs (e.g., Ramberg and Parsons, 2012). In fact, the so called “burner-tip parity” is based on the competitive prices where these fuels are used – at the burner tip (Brown and Yücel, 2008, p. 48).⁵ Both simple rules of thumb and burner-tip parities imply a strong long-run relationship between oil and gas prices.

Another strand of the literature, on the other hand, argues that the relationship between oil and gas prices observed in the past stems from oil indexation and will disappear in the absence of indexation as supply and demand dynamics of these two types of fuel are fundamentally different from each other. For example Stern and Rogers (2012) argued that the combination of several factors such as the eventual elimination of oil products from many stationary energy sectors, the cost and inconvenience of oil burning equipment, the emergence of modern gas-burning equipment, and stringent environmental regulations reduced the possibility of substitution between gas and oil products, and hence rendered the oil-gas price relationship dubious. As oil and gas markets have different fundamental driving factors, gas prices are unlikely to recouple again with oil prices (e.g., Stern, 2014; Zhang et al., 2018).

The empirical relationship between crude oil and natural gas prices has usually been examined using co-integration tests. Earlier research

produced results supporting the existence of a long-run relationship between the two prices. For example, Yücel and Guo (1994) found that a 1% change in oil prices leads to a 0.89% change in natural gas prices in the long run. As the estimated long-run elasticity coefficient was close to unity, they concluded that oil and natural gas are relatively good substitutes. Brown and Yücel (2008) also estimated the long run elasticity coefficient to be around 0.9. While they found stable long run relationship between the crude oil and natural gas prices, they concluded that the simple rules of thumbs cannot explain differential movements, particularly complex short-run dynamics. In particular, they found that both demand (weather conditions) and supply (storage and production shortages) conditions have significant effect on relative prices in the short run. Bachmeier and Griffin (2006) also found a long-run equilibrium relationship between crude oil and natural gas prices. However, they concluded that the two fuels are not close substitutes as the estimated long-run co-integrating coefficient was below the expected levels. They also found little day-to-day co-movement in prices. Hartley et al. (2008) argued that “the relationship between these two commodities is indirect, acting via competition between natural gas and residual fuel oil.” Similarly, Hartley and Medlock (2014) underlined the role of electricity generation technology in establishing the crude oil-natural gas relative prices and found that monetary and fiscal policy can affect the long-term movements in the relative prices. Jadidzadeh and Serletis (2017), on the other hand, instead of analysing the effect of the aggregate oil prices shocks on gas prices, disentangled oil supply shocks, oil-specific demand shocks and aggregate demand shocks driven by the global business cycle, and found that the response of the gas prices vary depending on the cause of the oil price shock. Overall, they concluded that real gas prices decouple from real oil prices only episodically.

Some researchers argued that the long-run co-movement of prices depends on the state of market regulations or integration with other markets. For example, Asche et al. (2006) found that a combination of deregulation and autarky may lead to market integration. In particular, they analysed the UK energy market and found a long-run stable relationship among crude oil, natural gas and electricity prices during the 1995–1998 period when the UK gas market was liberalized but not connected to the European grid. However, when the interconnector pipeline connecting the UK and the continental European gas grids became operational, the UK energy market became separated into three different market segments whereas the UK gas prices converged to the European prices. Serletis and Rangel-Ruiz (2004), on the other hand, argued that the deregulation of the market leads to decoupling of crude oil and natural gas prices. In particular, they analysed the strength of shared trends and cycles between the crude oil (WTI) and natural gas (Henry Hub) prices. They found no common and co-dependent cycles, and concluded that deregulation of the oil and gas markets in the US lead to decoupling of these two prices. However, they found a common synchronized cycle between the Henry Hub and AECO Alberta gas prices, providing an evidence of integration of the US and Canadian gas markets.

Usefulness of the conventional methods has increasingly been questioned in the empirical literature. Some authors have suggested that the relationship between the two prices might have changed over time arguing that several factors such as technological progress, productivity shocks, severe economic crisis, changes in preferences or policy changes may cause the equilibrium relationship to change over time. In fact, observed decoupling of oil and gas prices in the North American markets was usually attributed to shale gas boom (e.g., Wakamatsu and Aruga, 2013; Caporin and Fontini, 2017; Zhang and Ji, 2018). To allow for time-varying relationship between the prices, Erdős (2012) split the data to pre- and post-2009 period and found that while the US gas price has decoupled from crude oil price after 2009, the UK gas price remained linked to oil prices. Caporin and Fontini (2017) also analysed the oil-gas price relationship for different sub-periods. Although they found a break in the US oil-gas price relationship, they concluded that “it is not unequivocally possible to assess whether or not a new long-run

⁴ Another simple rule of thumb is the 10-to-1 rule under which the gas price (per MMBtu) is one tenth of crude oil price (per barrel).

⁵ There are two burner-tip parity rules: one based on competition between natural gas and residual fuel oil, the other on competition with distillate fuel oil. Both burner-tip parity rules convert a crude oil price to a fuel price, and relate it back to hub price, thus implying a linear relationship between gas and oil prices (see Brown and Yücel, 2008).

relationship between oil and gas has been established". Zhang and Ji (2018) used rolling windows to estimate time-varying order of integration of relative prices across three major gas markets, i.e., the US, European and Japan markets, and found that the persistence of relative prices have risen significantly in all three markets. Their results suggest that while the observed decoupling in the oil and gas price relationship was temporary in the European and Asian markets, gas prices decoupled permanently from crude oil prices in the US. Batten et al. (2017) also found that the dynamic linkages between crude oil and natural gas prices have changed significantly over time using rolling windows.

Other researchers, instead of sub-period decomposition, applied formal statistical procedures to test whether the relationship between crude oil and natural gas prices changed over time. For instance, Wakamatsu and Aruga (2013) applied structural break tests that test the breaks and produce break dates endogenously, and found strong evidence supporting multiple breaks in the relationship among oil price and gas price relationship as well as gas consumption in the US and Japan. They also found that the failure to take account of structural breaks leads to biased results. Similarly, Chevallier and Ielpo (2013), using co-integration tests under structural breaks, found strong evidence in favour of existence of the long run relationship between crude oil and natural gas prices as well as between oil and petroleum products' prices. More recently, Szafranek and Rubaszek (2023) used a time-varying structural VAR model and found that that prices are decoupled in the US. On the other hand, their results suggest that the decoupling seems to be short lived and gas prices remain strongly linked to oil prices in the European market.

Yet another strand of the literature have emphasised possible nonlinearities in the oil-gas price linkages. Several factors such as adjustment costs, imperfect competition and other market rigidities may cause to nonlinearities in price responses. In fact, the oil-gas price linkage is usually justified by close substitutability of oil products and natural gas. If the relative prices deviate from the equilibrium level, cost minimization behaviour of agents will trigger replacement of expensive fuel with cheaper substitute and hence, the demand and price of the substitute fuel will rise accordingly. This implies that relative prices can't deviate from the equilibrium level permanently. However, if the price difference is not big enough so that the gains from fuel switch do not cover the cost of equipment replacement, consumers will not switch to the substitute fuel, and hence the small deviations will persist. On the other, if the deviation from the equilibrium level is big enough to cover the replacement costs, agents will substitute fuels and hence, prices will return to the equilibrium level. This implies asymmetric response of price dynamics to price discrepancies. Similarly, strategic firm behaviour, inventory levels, production lags and market structure may trigger the so-called "rockets and feathers" effect, i.e., faster price adjustments to oil price increases than to oil price decreases (see also discussions in Atil et al., 2014, and references therein).

Asche et al. (2017) used Markov switching model to analyse varying gas pricing dynamics during peak and off-peak periods arguing that gas pricing may vary across seasons. They found that gas prices in the UK tend to decouple from oil prices during peak demand periods (i.e., fall and winter seasons), throughout which gas prices become more volatile and tend to follow a random walk process. During off-peak periods, on the other hand, gas prices seem to be integrated with oil prices, and the law of one price holds. Brigida (2014) also used Markov switching models to analyse shifts in the long run relationship between Henry Hub and WTI prices. He found co-integrating relationship only after allowing for regime switching dynamics. He also found that the failure to take account of possible nonlinearities in the long run level relationship will also lead to poor estimates of the short run relationships. On the other hand, Geng et al. (2016) using Markov regime-switching model found that Henry Hub prices have decoupled from WTI prices, while NBP and Brent prices have continued to exhibit a long-term equilibrium relationship. Another strand of the literature argued that the relationship between prices may vary depending upon the direction of price changes.

In fact, Atil et al. (2014) used nonlinear autoregressive distributed lag model (NARDL) to analyse asymmetric effects of negative versus positive oil price changes on gasoline and natural gas prices in the US market, and found significant nonlinearities both in the short and long runs for both commodities. Batten et al. (2017) also tested for asymmetric causality between prices and found strong asymmetries in the effects of positive versus negative oil price changes on natural gas prices.

Our approach in this paper in analysing the relationship between the crude oil and natural gas prices differ from the existing literature in several ways. First, while both nonlinearities and structural shifts have been examined extensively in the literature as briefly discussed above, the possibility of time-varying nonlinearity is not examined yet. To the best of our knowledge, only Batten et al. (2017) examined possible asymmetries in the predictive power of positive versus negative price changes and found some evidence supporting time-varying causality. In particular, they used rolling windows to examine Granger-causality of positive versus negative price changes, and found causality only in some sub-periods. Although they found some time-varying asymmetries in the predictive power of positive versus negative price changes, they did not test the statistical significance of breaks in the estimated relationships. Moreover, they also did not estimate and report the effects of negative and positive price changes. They also did not test statistical significance of the difference between the effects of positive versus negative price changes. Unlike Batten et al. (2017), we estimate and test significance of nonlinear and time varying effects of oil prices on gas prices. We also estimate time-varying and nonlinear effects of the demand and supply conditions on the short-run gas prices.

Second, unlike previous researchers, we do not impose specific form of breaks and nonlinearities, but use statistical tests to choose the most appropriate model to describe the relationship between variables under investigation. While Erdős (2012), Caporin and Fontini (2017), Zhang and Ji (2018) and Batten et al. (2017) provided evidence of time-varying relationship among prices, they did not test whether the observed differences in the estimated coefficients are statistically significant. Only Wakamatsu and Aruga (2013) tested significance of the breaks whereas Chevallier and Ielpo (2013) applied a co-integration test robust to structural breaks. However, their methodology impose instantaneous changes in the regression parameters. Similarly, Atil et al. (2014) and Batten et al. (2017) imposed threshold type nonlinearity whereas Asche et al. (2017) and Geng et al. (2016) impose Markov regime switching model and do not pre-test imposed nonlinear structure. Only Brigida (2014) used some statistical tests to compare adequacy of the imposed structures and found that a two-regime model is preferred to a single regime (linear) and three-regime models. Unlike the existing literature, we test statistical significance of both structural shifts and nonlinearities in the relationship between variables.

Third, in addition to long-run relationship, we also examined possible nonlinearities and structural shifts in the short-run dynamics as well. Previous researchers used nonlinear and structural break models to examine only the long run equilibrium relationship among variables but used conventional linear models to analyse short-run dynamics. Contrary to the previous researchers, we find that short run dynamics are also highly nonlinear and vary significantly over time apart from the long-run relationship. All in all, we find that the relationship between crude oil and natural gas prices are more complex than previously thought.

3. Methodology

The existence of a long-run relationship among economic variables has usually been investigated by co-integration tests. Consider the following simple relationship between oil and gas prices:

$$gp_t = \alpha + \beta op_t + \varepsilon_t \quad (1)$$

where gp_t is (the natural log of) gas prices, op_t is (the natural log of) oil

prices, α and β are parameters to be estimated and ε_t is random disturbances. While both op_t and gp_t might be non-stationary processes, if the disturbances ε_t follow a stationary process, op_t and gp_t are said to be cointegrated. In this case, the two price series share a common stochastic trend and the Eq. (1) defines the long-run equilibrium relationship between oil and gas prices (Engle and Granger, 1987). On the other hand, if the disturbances ε_t are not stationary, the observed correlation between the series are spurious and there is no genuine relationship between oil and gas prices.

If the series are cointegrated, then the parameter β measures the long-run response of gas prices to changes in oil prices (i.e., oil price elasticity) whereas α is the (log of) proportionality coefficient. Any deviations from the equilibrium relationship (i.e, ε_t) are short lived and variables will change in a way to correct such deviations. Short-run dynamics of the variables are usually modelled by an error-correction model (ECM):

$$\Delta gp_t = \mu + \gamma ECT_{t-1} + \sum_{i=1}^k \delta_i \Delta gp_{t-i} + \sum_{j=0}^m \eta_j \Delta op_{t-j} + \Omega' v_t + u_t \quad (2)$$

where the error correction term ECT_{t-1} is the lagged error term ε_{t-1} in Eq. (1). The parameter γ measures the speed of adjustment of deviations towards the long-run equilibrium relationship, v_t is a vector of exogenous variables that have only short-run effects on gas prices, Ω and η_j are short-run effects of exogenous variables and oil prices, respectively, on gas prices.

The long- and short-run relationships between gas prices and oil prices given in Eqs. (1) and (2) assume that the relationship is linear both in the long- and short-runs. However, as briefly discussed above, the relationship between the prices might inherently be nonlinear and vary significantly over time. Therefore, in modelling the relationship between crude oil and natural gas prices, we use time varying smooth transition regression models (TV-STR) proposed by Lundbergh et al. (2003). The TV-STR models nests linear, nonlinear, structural change, and time-varying nonlinear models as special cases. Model specification tests allow researchers to choose which alternative is more appropriate for the relationship under investigation. Below we discuss main features and the specification procedure of the TV-STR models.

3.1. The TV-STR models

A time-varying smooth transition regression model can be written as follows:

$$y_t = [\phi'_1 x_t (1 - F(s_t)) + \phi'_2 x_t F(s_t)] [1 - G(t)] + [\phi'_3 x_t (1 - F(s_t)) + \phi'_4 x_t F(s_t)] G(t) + \varepsilon_t \quad (3)$$

where $y_t = gp_t$ and $x_t = (1, op_t)'$ when we analyse the long-run equilibrium relationship, whereas $y_t = \Delta gp_t$ and $x_t = (1, ECT_{t-1}, \Delta gp_{t-1}, \dots, \Delta gp_{t-k}, \Delta op_t, \Delta op_{t-1}, \dots, \Delta gp_{t-m}, v_t)'$ when we analyse the short-run dynamics.

The two transition functions $F(s_t)$ and $G(t)$ capture nonlinearities and the structural shift, respectively. These transition functions are continuous functions bounded between 0 and 1. Two popular specifications of the regime-switching function $F(s_t)$ are logistic and exponential functions given in Eqs. (4a) and (4b):

$$F(s_t) = [1 + \exp\{-\theta_1(s_t - c)\}]^{-1}, \theta_1 > 0 \quad (4a)$$

$$F(s_t) = 1 - \exp\{-\theta_1(s_t - c)^2\}, \theta_1 > 0 \quad (4b)$$

where s_t is the so-called switching variable that governs the gradual transitions between the two regimes corresponding to the extreme values of the regime-switching function, $F(s_t) = 0$ and $F(s_t) = 1$. The parameter θ_1 determines the smoothness of transition between the re-

gimes and the parameter c is the threshold parameter separating the regimes. The extreme values of the logistic function given in Eq. (4a) are associated with the sign of deviations of the switching variable s_t from the threshold parameter c . Therefore, the logistic regime-switching function is convenient for modelling asymmetric dynamics depending on the direction of deviations. On the other hand, the extreme values of the exponential function (4b) are associated with the size of the deviations ($s_t - c$) irrespective of the sign. Hence, the exponential function (4b) is more appropriate for modelling asymmetric responses to big versus small deviations.

Structural break is also modelled using smooth transition functions. While Lundbergh et al. (2003) considered only a logistic transition function, we follow Lin and Teräsvirta (1994) who proposed three different specifications. Particularly, they considered the following specifications for the transition function $G(t)$:

$$G(t) = [1 + \exp\{-\theta_2(t - c)\}]^{-1}, \theta_2 > 0 \quad (5a)$$

$$G(t) = 1 - \exp\{-\theta_2(t - c)^2\}, \theta_2 > 0 \quad (5b)$$

$$G(t) = [1 + \exp\{-\theta_2(t^3 + c_1 t^2 + c_2 t + c_3)\}]^{-1}, \theta_2 > 0 \quad (5c)$$

The transition functions $G(t)$ are also bounded between 0 and 1, each corresponding to different regimes. In all three specifications, the smoothness parameter θ_2 determines the speed of the structural change. All three specifications use time t as the switching variable although each of the functions depict rather different structural change patterns. The logistic function $G(t)$ given in Eq. (5a) is monotonic function of t and the threshold parameter c represents (the midpoint of) the structural break. As the value of the logistic function gradually changes from 0 to 1, the logistic function $G(t)$ given in Eq. (5a) is convenient for modelling permanent structural changes. On the other hand, in the exponential function given in Eq. (5b), $G(t) \rightarrow 1$ both for $t \rightarrow 0$ and $t \rightarrow +\infty$, but $G(t) \rightarrow 0$ as $t \rightarrow c$, implying that the exponential function is convenient for modelling temporary breaks. However, for the third order logistic function given in Eq. (5c), the transition between the extreme values of the function is no longer monotonic, which allows for modelling rather different types of structural breaks. The third order logistic function also implies that the break is permanent.

A nice feature of the TV-STR model given in Eq. (3) with transition functions (4a)-(4b) and (5a)-(5c) is that, this model nests a linear, a regime-switching, and a gradual break models as special cases. In particular, when $\phi'_1 = \phi'_2 = \phi'_3 = \phi'_4 = \phi'$, the Eq. (3) collapses to

$$y_t = \phi' x_t + \varepsilon_t \quad (6)$$

which is the simple linear model given in Eqs. (1) or (2). When $\phi'_1 \neq \phi'_2, \phi'_1 = \phi'_3, \phi'_2 = \phi'_4$ the model reduces to

$$y_t = \phi'_1 x_t (1 - F(s_t)) + \phi'_2 x_t F(s_t) + \varepsilon_t \quad (7)$$

which gives rise to a smooth transition regression (STR) model which is used to model regime-switching relationships. On the other hand, if $\phi'_1 = \phi'_2, \phi'_3 = \phi'_4$, and $\phi'_1 \neq \phi'_3$ one obtains the following time-varying (TV) regression model:

$$y_t = \phi'_1 x_t (1 - G(t)) + \phi'_3 x_t G(t) + \varepsilon_t \quad (8)$$

which allows for gradual shift in the parameters of the model over time but no regime-wise asymmetries. Also note that when $\theta_i \rightarrow +\infty$, for $i = 1, 2$ the transition between the extreme values of transition functions given in (4a)-(4b) and (5a)-(5c) occurs almost instantaneously, which implies a sharp break in the parameters of the model. Thus, the smooth transition functions nests abrupt change models as special cases. On the other hand, when $\theta_i \rightarrow 0$, for $i = 1, 2$ the transitions occurs very slowly and in the extreme case $\theta_1 = \theta_2 = 0$, the model collapses to a linear model. Also note that when $\theta_1 \neq 0$ and $\theta_2 = 0$ one obtains a STR model and a TV

model when $\theta_1 = 0$ and $\theta_2 \neq 0$.

3.2. Specification of TV-STR models

As Lundbergh et al. (2003) point out, another nice feature of the TV-STR modelling approach is that one may use statistical tests to decide whether the full TV-STR model is required to describe the relationship between the variables or a sub-model such as a TV or a STR model is more appropriate for the data under consideration. One may adopt either a specific-to-general modelling strategy or a specific-to-general-to-specific modelling strategy to choose the most appropriate specification. As the latter strategy does not require nonlinear estimation in the specification stage, we use this modelling strategy. Also, simulation results of Lundbergh et al. (2003) suggested that this modelling approach has better small sample properties for highly nonlinear DGPs.

The specific-to-general-to-specific modelling strategy consists of three steps. In the first step, one needs to specify an appropriate linear model for the series under investigation. In the second step, one tests linearity directly against a TV-STR alternative. If the linearity is rejected, in the third step one tests sub-hypotheses to decide whether the full TV-STR model is required or a nested sub-model such as a TV or a STR model is sufficient to describe the relationship.

Linearity tests are based on the following auxiliary regression equation obtained by replacing the transition functions by their first order Taylor expansion⁶:

$$y_t = \psi'_1 x_t + \psi'_2 \tilde{x}_t s_t + \psi'_3 \tilde{x}_t t + \psi'_4 \tilde{x}_t s_t t + e_t \tag{9}$$

where $\tilde{x}_t = op_{t-1}$ or $\tilde{x}_t = (ECT_{t-1}, \Delta gp_{t-1}, \dots, \Delta gp_{t-k}, \Delta op_t, \Delta op_{t-1}, \dots, \Delta gp_{t-m}, v_t)'$, ψ_i , $i = 1, \dots, 4$ are functions of ϕ'_i , θ_j , c_k for $i = 1, \dots, 4$, $j = 1, 2$, and $k = 1, 2, 3$ given in Eq. (3) – (5), and e_t includes residuals ε_t as well as the reminder terms from the two Taylor series approximation.

The relationship between coefficients of Eq. (9) and the TV-STR model given in Eq. (3) – (5) implies that $\psi'_2 = \psi'_3 = \psi'_4 = 0$ if and only if $\theta_1 = \theta_2 = 0$. Also notice that Eq. (9) collapses to a linear model when $\psi'_2 = \psi'_3 = \psi'_4 = 0$. Therefore, the null hypothesis of linearity ($H_0 : \theta_1 = \theta_2 = 0$) can alternatively be written as $H_0^{TV-STR} : \psi'_2 = \psi'_3 = \psi'_4 = 0$. Rejection of the null hypothesis H_0^{TV-STR} suggests TV-STR-type nonlinearity in the relationship between variables. If the null H_0^{TV-STR} is rejected, one can test nested sub-hypotheses $H_0^{STR} : \psi'_2 = \psi'_4 = 0$ and $H_0^{TV} : \psi'_3 = \psi'_4 = 0$ two decide whether a full TV-STR model is required or a time varying or a nonlinear specification is sufficient to describe the relationship between the variables. If both H_0^{STR} and H_0^{TV} are rejected, one must chose the TV-STR model. If H_0^{STR} is rejected but H_0^{TV} is not, one must chose a regime-switching STR model given in Eq. (7). On the other hand, if H_0^{TV} is rejected but H_0^{STR} is not, one must decide on a TV model given in Eq. (8).⁷

As the level of oil and gas price series contain a stochastic trend, conventional procedures for testing linearity might not be appropriate for testing linearity against the TV-STR type nonlinearity. In fact, linearity tests are based on stationarity of the series (e.g., Teräsvirta, 1994). Therefore, while the auxiliary regression Eq. (9) can be used to test linearity for the short-run relationship between gas and oil prices, one needs a different approach for testing linearity in a co-integration

⁶ Testing linearity directly against the TV-STR type nonlinearity is complicated by the presence of nuisance parameters under the null hypothesis of linearity. Researches usually use linear approximations of transition functions to overcome this problem. See, e.g., Teräsvirta (1994) and Luukkonen et al. (1988) for details.

⁷ If neither H_0^{TV} nor H_0^{STR} is rejected but the more general null hypothesis H_0^{TV-STR} is rejected, one must resort to “specific-to-general” specification procedure to decide whether a TV or a STR model is more appropriate for the data.

relationship. Choi and Saikkonen (2004) suggests augmenting the test equation with leads and lags of the first difference of dependent variables.⁸ In particular, linearity of the long-run relationship between the levels of gas prices and oil prices can be tested using the following auxiliary regression model:

$$gp_t = \alpha_0 + \beta_0 op_t + \alpha_1 s_t + \beta_1 op_t s_t + \alpha_2 t + \beta_2 op_t t + \alpha_3 s_t t + \beta_3 op_t s_t t + \sum_{j=-K}^K \pi_j \Delta op_{t-j} + e_t \tag{9a}$$

A nice feature of this approach is that it does not rely on the assumption of exogeneity of the regressors and permits both serial and contemporaneous correlations between the regressors and the error term.

The appropriate switching variable s_t is not known a priori. Therefore, the H_0^{TV-STR} hypothesis can be tested for several candidate switching variables, and the one with the lowest p -value can be chosen. This approach allows one to choose the most appropriate transition variable that governs regime changes in the relationships. After selecting the switching variable one must decide on the type of the transition function. In order to select the appropriate form of the transition functions, following Teräsvirta (1994) and Lin and Teräsvirta (1994), we replace the transition functions with their third order Taylor approximations. In particular, we consider the following auxiliary regression model to choose the type of the transition function $F(s_t)$ ⁹:

$$y_t = \zeta'_0 \tilde{x}_t + \zeta'_1 \tilde{x}_t s_t + \zeta'_2 \tilde{x}_t s_t^2 + \zeta'_3 \tilde{x}_t s_t^3 + e_t \tag{10}$$

where $\tilde{x}_t = (x_t, tx_t)'$. Using auxiliary regression model given in Eq. (10) we test the following null hypotheses:

$$\begin{aligned} H_{03}^{STR} : \zeta'_3 &= 0 \\ H_{02}^{STR} : \zeta'_2 &= 0 | \zeta'_3 = 0 \\ H_{01}^{STR} : \zeta'_1 &= 0 | \zeta'_3 = \zeta'_2 = 0 \end{aligned}$$

The decision rule is as follows: if the p -value corresponding to H_{02}^{STR} is the smallest, choose the exponential transition function. Otherwise, choose the logistic transition function. In order to choose the type of the transition function governing the structural break, we estimate the auxiliary regression (11):

$$y_t = \theta'_0 \tilde{x}_t + \theta'_1 \tilde{x}_t t + \theta'_2 \tilde{x}_t t^2 + \theta'_3 \tilde{x}_t t^3 + e_t \tag{11}$$

with $\tilde{x}_t = (x_t, s_t x_t)'$ and test the following null hypotheses:

$$\begin{aligned} H_{03}^{TV} : \theta'_3 &= 0 \\ H_{02}^{TV} : \theta'_2 &= 0 | \theta'_3 = 0 \\ H_{01}^{TV} : \theta'_1 &= 0 | \theta'_3 = \theta'_2 = 0 \end{aligned}$$

If the H_{03}^{TV} hypothesis is rejected, we chose a third-order logistic transition function. If H_{03}^{TV} is not rejected, we test H_{02}^{TV} . If this hypothesis is rejected, we chose an exponential transition function. If neither H_{03}^{TV} nor H_{02}^{TV} is rejected, we test H_{01}^{TV} which is a test of parameter constancy. Notice that non-rejection of the H_{01}^{TV} implies that there is no structural shift in the relationship between the variables.

⁸ We included the same augmentation terms in estimating the long-run level relationship as suggested by Saikkonen and Choi (2004).

⁹ In order to save degrees of freedom in the auxiliary regression model we used a first order Taylor approximation for the transition function $G(t)$ that governs the structural break.

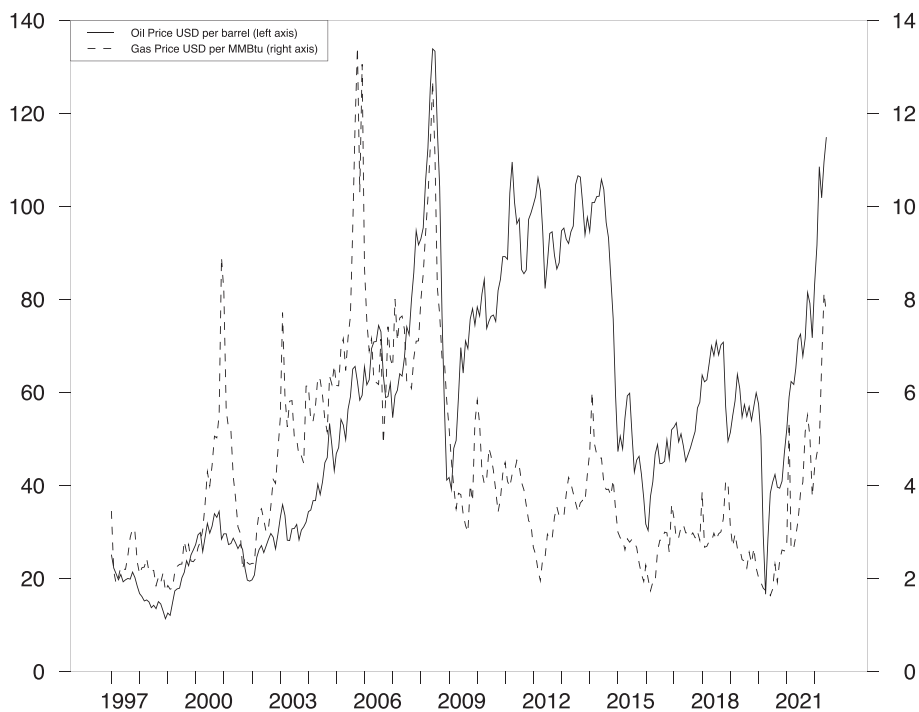


Fig. 1. Oil and gas prices.

4. Data and estimation results

In this paper we focus on the US gas market, which is the most mature and most developed gas market. We use monthly observations covering the 1997:01–2022:06 period.¹⁰ Gas prices are Henry Hub natural gas spot prices (Dollars per million Btu), oil prices are Cushing, Oklahoma WTI spot price (Dollars per barrel). Fig. 1 below plot time series graphs of both price series.¹¹

In addition to long-run equilibrium relationship, we also analyse whether short-run relationship between the crude oil and gas prices exhibits nonlinearities and varies over time. It has been well established that simple models relating natural gas prices to crude oil prices do not explain a significant share of short-run gas price movements (see, for example, discussions in Brown and Yücel, 2008; Ramberg and Parsons, 2012). As omitting relevant explanatory variables would lead to model misspecification and biased estimates, following the previous literature, we also use the number of heating degree days, the deviation from the normal (prior 10-year average) number of heating degree days, the number of cooling degree days, the deviation from the normal (prior 10-year average) number of cooling degree days, the deviation of the amount of natural gas storage from its average, and Baker Hughes'

¹⁰ The period and frequency of the data are dictated by the availability of exogenous control variables used in the error correction model to analyse the short run relationship between gas and oil prices. Additionally, as most of the existing studies also use monthly data, our results can be easily compared to the vast majority of the empirical research into oil and gas price relationship.

¹¹ To see whether our results are robust or depend on the choice of crude oil (WTI) and natural gas (Henry Hub) price series used in this paper, we carried out several experiments. Although WTI is the main oil benchmark for the American market as known, we used the Brent prices in our analysis as well. All the specification test results as well as estimates of the equilibrium relationship were almost the same that are reported in this paper. In addition, we also used gas price series for the European and Far Eastern markets (data were obtained from the World Bank Commodity Price ("Pink Sheet") Data) and found that the equilibrium relationships between crude oil (Brent) and regional gas prices are also highly nonlinear and varies over time. These results are available upon request.

North American rotary rig count data as control exogenous variables in modelling the short-run gas and oil price relationship. All the data, except for rig counts, which are taken from Baker Hughes, are taken from the US Energy Information Administration database.

4.1. Stationarity and co-integration tests

In order to build the most appropriate model one needs to know the order of integration of the data. We take natural logarithm of crude oil price, natural gas price and rig count data before statistical analyses. We use conventional ADF, PP and KPSS tests to examine stochastic properties of the variables used in the empirical analysis. Results of these tests, that are presented below in Table 1, suggest that the crude oil price, natural gas price and rig count series are I(1) processes whereas climate-related and gas storage data are I(0).

As both price series are integrated of order one, we proceed to test whether the series are co-integrated. For this purpose, we first apply the conventional Johansen-Juselius test (Johansen, 1991; Johansen and Juselius, 1990) and Engle-Granger (EG) (Engle and Granger, 1987) co-integration tests, results of which are presented below in Table 2.

Neither the Johansen-Juselius nor the Engle-Granger tests suggest co-integration between the oil and gas prices, implying no long-run relationship between the price series. However, it is well-known that the failure of taking account of possible structural breaks may lead to misleading results. In fact, visual inspection of the residuals from the regression of gas prices on oil prices (see Fig. 2) show a clear shift in the mean of the series around 2008–2010. In his seminal contribution, Perron (1989) has shown that power of conventional stationarity tests decrease significantly if there is a structural change, even if the data is indeed stationary. Therefore, to test whether the residuals from the regression of gas prices on oil prices follow a stationary process, we also applied stationarity tests proposed by Leybourne et al. (1998), Sollis (2004) and Omay et al. (2018) that allow for gradual shift in the deterministic components of the series. In addition to structural shift, unit root test procedures of Sollis (2004) and Omay et al. (2018) allow for nonlinearity in the adjustment towards the gradually changing mean. All these tests use logistic transition functions to model gradually changing trend in the series. Results of these tests are reported in

Table 1
Stationarity test results.

Series	ADF	PP	KPSS
Level of the series			
Oil prices ^a	-2.628(1)	-2.243(1)	2.590(1)
Gas prices ^b	-2.576(0)	-2.585(0)	4.351(0)
Heating degree days ^b	-3.103(12)	-3.357(12)	0.248
	**	**	(12)***
Heating degree days deviation from normal ^b	-4.956	-15.087	0.079
	(12)***	(12)***	(12)***
Cooling degree days ^b	-2.994(13)	-3.841	0.328
	**	(13)***	(13)***
Cooling degree days deviation from normal ^b	-13.307	-13.350	0.057
	(0)***	(0)***	(0)***
Natural gas storage deviation from normal ^b	-5.061	-3.727	0.134
	(11)***	(11)***	(11)***
Rig count ^b	-2.686(3)	-1.834(3)	1.415(0)
First differences			
Oil prices ^b	-13.417	-13.462	0.066
	(0)***	(0)***	(0)***
Gas prices ^b	-12.647	-18.709	0.061
	(0)***	(0)***	(0)***
Rig count ^b	-6.300	-6.396(2)***	0.120
	(2)***		(2)***

Number of optimal lag length is chosen so as to remove any significant residual autocorrelation and shown in parenthesis. Linear trend is included in the test equation if was statistically significant.

^a Test equation includes a constant and a linear trend.

^b Test equation includes a constant only.

*** Denotes rejection of the null hypothesis of unit root at 1% significance level for the ADF and PP tests and no rejection of the null hypothesis of stationarity at 99% confidence level for the KPSS test.

Table 3.

All three tests suggest that the residuals from the regression of natural gas prices on oil prices follow a stationary process once one accounts for possible structural breaks. As the residuals are stationary, we conclude that the price series are cointegrated, implying the existence of a long-run statistical equilibrium relationship whereas the relationship might have shifted during the analysed period.

4.2. Estimating the long run equilibrium level relationship

The next step in the estimation of the long-run relationship between gas prices and oil prices is testing linearity of the long run level relationship. As we found stationarity in the residuals only after allowing for structural break, we test linearity of the cointegrating relationship against time varying smooth transition (TV-STR) type nonlinearity. Since monthly time series exhibit significant short-run fluctuations, we consider annual changes in price series as candidate transition variables that govern regime shifts (see also Lundbergh et al., 2003).¹² The results of the linearity tests are reported in the Table 4.

The null hypothesis of linearity is convincingly rejected for all candidate transition variables. As the null of linearity is rejected more strongly for the fourth lag of annual changes in gas prices ($\Delta_{12}gp_{t-4}$) we chose this variable as the switching variable that governs changes in regimes. Using this switching variable, we carry out specification tests to choose the appropriate form of the transition function. As the *p*-value

¹² We also tested linearity of the level relationship using (lags of) the long differences in the series, namely $\Delta_j x_t \equiv x_t - x_{t-j}$ as candidate transition variables. While the null hypothesis was rejected for all long difference series as well, it was more statistically strongly rejected for annual differences. In order to save space, we do not report results with long differences, which are available upon request.

Table 2
Linear co-integration test results.

Null hypothesis	JJ cointegration test				EG cointegration test	
	J_{trace}	5% c.v. ^a	J_{max}	5% c.v. ^a	τ	5% c.v. ^b
$r = 0$	11.484	15.495	7.752	14.265	-3.024	-3.356
$r \leq 1$	3.732	3.841	3.732	3.841		

J_{trace} and J_{max} denote the trace and maximum eigenvalue statistics, respectively.

^a MacKinnon et al. (1999) critical value at 5% significance level.

^b MacKinnon (2010) critical value at 5% significance level.

corresponding to H_{01}^{STR} is the smallest of H_{0i}^{STR} , $i = 1, 2, 3$, we chose the logistic transition function to model regime shifts. In addition, as the null hypothesis H_{03}^{TV} is rejected, we do not test H_{02}^{TV} and H_{01}^{TV} , and choose the third order logistic function to model structural change. Thus, the specification tests suggest the following long-run relationship between the oil and gas prices:

$$gp_t = [(\alpha_0 + \beta_0 op_t)(1 - F(\Delta_{12}gp_{t-4})) + (\alpha_1 + \beta_1 op_t)F(\Delta_{12}gp_{t-4})][1 - G(t)] + [(\alpha_2 + \beta_2 op_t)(1 - F(\Delta_{12}gp_{t-4})) + (\alpha_3 + \beta_3 op_t)F(\Delta_{12}gp_{t-4})]G(t) + \epsilon_t \tag{12}$$

with

$$F(\Delta_{12}gp_{t-4}) = [1 + \exp\{-\theta_1(\Delta_{12}gp_{t-4} - c)\}]^{-1} \tag{13}$$

$$G(t) = [1 + \exp\{-\theta_2(t^3 + c_1 t^2 + c_2 t + c_3)\}]^{-1} \tag{14}$$

Coefficient estimates of the long run equilibrium relationship are given below in Table 5.

Notice that the TV-STR model provides significant improvement over the linear model in terms of fit to the actual data. The estimated coefficients $\hat{\alpha}_i$ and $\hat{\beta}_i$, $i = 0, 1, 2, 3$ in the nonlinear TV-STR model can be construed as intercept and slope coefficients, respectively, for different regimes. For example, $\hat{\beta}_0$ and $\hat{\beta}_1$ ($\hat{\alpha}_0$ and $\hat{\alpha}_1$) are the slope (intercept) coefficients before the break (i.e. $\hat{G}(t) = 0$) whereas $\hat{\beta}_0$ ($\hat{\alpha}_0$) is the slope (intercept) for the periods associated with lower gas price changes ($F(\Delta_{12}gp_{t-4}) \rightarrow 0$) while $\hat{\beta}_1$ ($\hat{\alpha}_1$) is the slope (intercept) for the periods associated with higher price changes ($F(\Delta_{12}gp_{t-4}) \rightarrow 1$). Similarly, $\hat{\beta}_2$ and $\hat{\beta}_3$ ($\hat{\alpha}_2$ and $\hat{\alpha}_3$) are the slope (intercept) coefficients after the break (i.e. $\hat{G}(t) = 1$).

To give a better depiction of the nature of the transition functions we plot graphs of the estimated transition functions. As can be seen from the Fig. 3, the structural change started around mid-2005 and was completed by the mid-2011. The regime-switching transition function implies that the oil and gas price relationship was highly nonlinear both before and after the structural change whereas the switch between the regimes was rather slow.

While the estimated coefficients reported in Table 5 represent slope and intercept coefficients in extreme regimes, in effect, coefficients vary significantly between these extreme values as suggested by the graphs of the estimated transition functions. Therefore, we also calculate the time-varying slope and intercept coefficients as implied by the transition functions. Specifically, we calculate the implied time-varying coefficients as:

$$\widehat{slope}_t = [\hat{\beta}_0(1 - \hat{F}(\Delta_{12}gp_{t-4})) + \hat{\beta}_1 \hat{F}(\Delta_{12}gp_{t-4})][1 - \hat{G}(t)] + [\hat{\beta}_2(1 - \hat{F}(\Delta_{12}gp_{t-4})) + \hat{\beta}_3 \hat{F}(\Delta_{12}gp_{t-4})]\hat{G}(t) \tag{15}$$

$$\widehat{intercept}_t = [\hat{\alpha}_0(1 - \hat{F}(\Delta_{12}gp_{t-4})) + \hat{\alpha}_1 \hat{F}(\Delta_{12}gp_{t-4})][1 - \hat{G}(t)] + [\hat{\alpha}_2(1 - \hat{F}(\Delta_{12}gp_{t-4})) + \hat{\alpha}_3 \hat{F}(\Delta_{12}gp_{t-4})]\hat{G}(t) \tag{16}$$

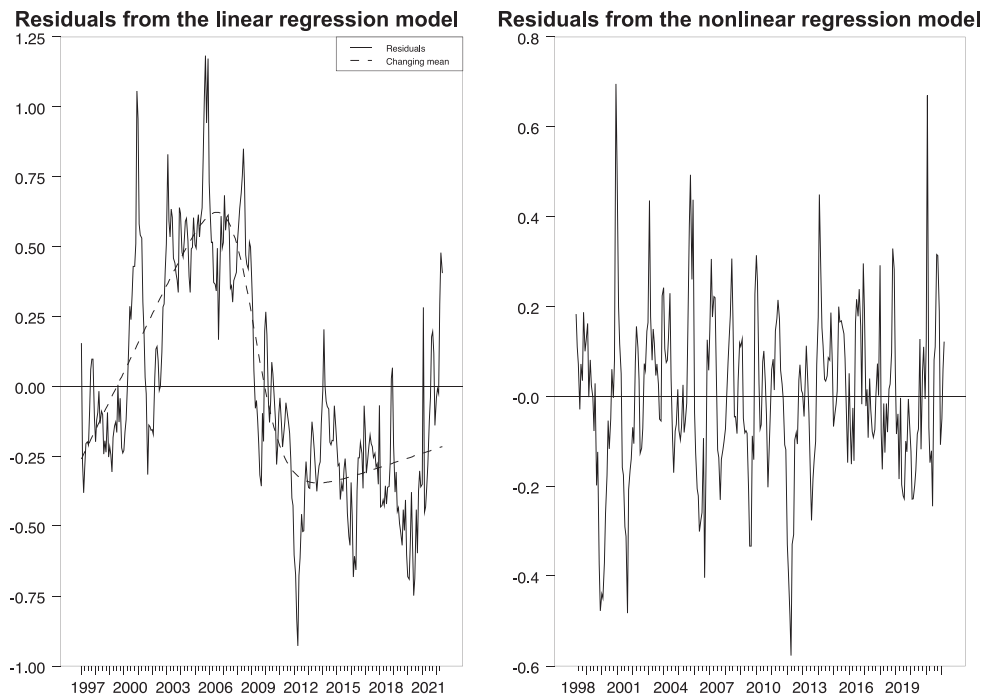


Fig. 2. Residuals from linear and nonlinear regression of gas prices on oil prices.

Table 3
Nonlinear co-integration test results.

	τ_L	τ_A	F_{AE}
Test statistic	-5.479***	-4.399***	15.988***
5% critical value	-4.867	-3.937	10.52
1% critical value	-5.435	-4.393	13.22

τ_L denote [Leybourne et al. \(1998\)](#) stationarity test that allows for linear adjustment towards gradually changing trend function.

τ_A denote [Sollis' \(2004\)](#) stationarity test that allows for threshold type asymmetric adjustment towards gradually changing trend function.

F_{AE} denote [Omay et al. \(2018\)](#) stationarity test that allows for asymmetric ESTAR (exponential smooth transition) type adjustment towards gradually changing trend function.

*** Denotes rejection of the null hypothesis of unit root at 1% significance level.

We present the graphs of the time varying coefficients below in [Fig. 4](#).¹³ As can readily be seen from the graph, the slope coefficient has fallen significantly after the break. Whereas the slope coefficient fluctuated around unity before 2005, it has fallen to 0.48 post 2011. Also notice that while the slope coefficient varied widely before the break, variation of the slope coefficient also has fallen after the break. Contrary to the slope coefficient, both the mean and variation of the intercept has risen significantly after the break. In fact, while the mean of the intercept coefficient was around -1.982 before the break, it has risen to -0.868 after the break.

The estimated model suggests that, in addition to the structural shift over time, the oil and gas price relationship is also asymmetric across regimes that are associated with (relatively big) gas price increases versus price falls as determined by the regime-switching function. To

¹³ The oil-price elasticity is the slope coefficient whereas the proportionality coefficient is the exponential of the intercept coefficient. The “before the break” period refers to the period prior to 2005:07 during which the transition function governing structural break was close to zero (i.e., $\hat{G}(t) \leq 0.05$). On the other hand, the “after the break” period covers period after 2011:05 during which $\hat{G}(t) \geq 0.95$.

examine whether the structural shift affected the regime-wise asymmetry or not, we test equality of coefficients across regimes before and after the break. Specifically, we test the null hypotheses $H_0 : \alpha_0 = \alpha_1$ and $\beta_0 = \beta_1$ and $H_0 : \alpha_2 = \alpha_3$ and $\beta_2 = \beta_3$. The associated test statistics are $\chi^2_2 = 5.361[0.069]$ and $\chi^2_2 = 2.958[0.228]$, respectively, providing statistical evidence in favour of regime-wise asymmetry before the break only. We also tested equality of coefficients before and after the break. In particular, we also tested $H_0 : \alpha_0 = \alpha_2$ and $\beta_0 = \beta_2$ and $H_0 : \alpha_1 = \alpha_3$ and $\beta_1 = \beta_3$ with test statistics $\chi^2_2 = 88.166[0.000]$ and $\chi^2_2 = 13.589[0.001]$, respectively. All in all, these tests provide additional statistical evidence supporting simultaneous asymmetry and structural shift in the crude oil and gas prices.

Estimates of the co-integrating relationship reveal interesting information about the nature of the long-run oil and gas price relationship. Although we find strong statistical evidence in favour of break in the oil-gas price relationship, the stationarity of the residuals implies the existence of long-run equilibrium relationship between the prices. This finding implies that the gas prices have not decoupled from oil prices permanently and a new relationship was established after the break, contradicting findings of [Erdős \(2012\)](#), [Caporin and Fontini \(2017\)](#), [Zhang and Ji \(2018\)](#), among others. Also, contrary to the findings of these researchers, our results imply that the shift in the equilibrium relationship was rather gradual, began in 2005 and ended in 2011.

Our results imply a significant shift in long-run equilibrium relationship. Although the long-run oil-price elasticity of gas prices fluctuated widely before the break it averaged unity, implying that any change in crude oil prices leads to an equally proportional changes in natural gas prices on average. After the break, both the mean and variation of the oil-price elasticity has fallen significantly. In fact, oil-price elasticity halved after the break, implying that one percentage increase/fall in crude oil prices causes a rise/fall in natural gas prices by a half of a percent. On the other hand, the mean and variance of the intercept coefficient increased after the break, suggesting that the oil and gas price relationship became less stable after the break. These results imply that equilibrium oil and gas price relationship became more volatile after the break whereas gas prices became less sensitive to oil prices.

In addition to structural change, we find asymmetry in the long-run equilibrium relationship between crude oil and gas prices as well. The

Table 4
Testing linearity of the long-run equilibrium relationship.

Candidate switching variable	Test statistic	p-value	Candidate switching variable	Test statistic	p-value
Panel A. Linearity tests against TV-STR type nonlinearity					
$\Delta_{12}op_{t-1}$	85.79***	4.17e-60	$\Delta_{12}gp_{t-1}$	108.37***	1.69e-69
$\Delta_{12}op_{t-2}$	88.39***	3.53e-61	$\Delta_{12}gp_{t-2}$	128.10***	1.62e-76
$\Delta_{12}op_{t-3}$	87.97***	6.66e-61	$\Delta_{12}gp_{t-3}$	136.92***	2.59e-79
$\Delta_{12}op_{t-4}$	87.70***	1.07e-60	$\Delta_{12}gp_{t-4}$	151.57***	1.01e-83
$\Delta_{12}op_{t-5}$	82.92***	1.81e-58	$\Delta_{12}gp_{t-5}$	125.79***	2.41e-75
$\Delta_{12}op_{t-6}$	79.02***	1.39e-56	$\Delta_{12}gp_{t-6}$	110.37***	1.05e-69
$\Delta_{12}op_{t-7}$	74.57***	2.20e-54	$\Delta_{12}gp_{t-7}$	94.06***	3.80e-63
$\Delta_{12}op_{t-8}$	72.59***	2.42e-53	$\Delta_{12}gp_{t-8}$	85.88***	1.54e-59
$\Delta_{12}op_{t-9}$	69.32***	1.22e-51	$\Delta_{12}gp_{t-9}$	72.80***	2.27e-53
$\Delta_{12}op_{t-10}$	65.89***	8.06e-50	$\Delta_{12}gp_{t-10}$	64.83***	2.86e-49
$\Delta_{12}op_{t-11}$	62.01***	1.03e-47	$\Delta_{12}gp_{t-11}$	59.04***	4.26e-46
$\Delta_{12}op_{t-12}$	64.60***	5.13e-49	$\Delta_{12}gp_{t-12}$	56.15***	2.01e-44
Panel B. Transition function specification tests					
	Test statistic	p-value		Test statistic	p-value
H_{03}^{STR}	2.476 **	0.045	H_{03}^{TV}	61.325***	1.38e-36
H_{02}^{STR}	3.967***	0.004			
H_{01}^{STR}	69.551***	1.71e-40			

$\Delta_{12}x_t \equiv x_t - x_{t-12}$.

*** and ** denote rejection of the null hypothesis of linearity at 1% and 5% significance levels, respectively.

We use the F-variant of the linearity tests that provides better small-sample properties. See, for example, Teräsvirta (1994) and Lundbergh et al. (2003). Boldface denotes selected transition variable.

Table 5
Estimates of coefficients of the long-run equilibrium relationship.

Linear model			Nonlinear model		
Coefficient	Coefficient Estimate	Standard Error (p-value)	Coefficient	Coefficient Estimate	Standard Error (p-value)
α	0.942	0.051 (0.000)	α_0	-2.435	0.315 (0.000)
β	0.007	0.001 (0.000)	β_0	1.095	0.092 (0.000)
			α_1	-1.281	0.672 (0.057)
			β_1	0.876	0.182 (0.000)
			α_2	-1.305	0.417 (0.002)
			β_2	0.535	0.095 (0.000)
			α_3	0.094	1.051 (0.929)
			β_3	0.347	0.205 (0.091)
			θ_1	2.337	1.484 (0.115)
			c	0.375	0.342 (0.272)
			θ_2	1039.721	743.176 (0.162)
			c_1	-1.319	0.061 (0.000)
			c_2	0.592	0.048 (0.000)
			c_3	-0.091	0.010 (0.000)
\bar{R}^2	0.172		\bar{R}^2	0.821	
SEE	0.417		SEE	0.192	
Log Likelihood	-165.482		Log Likelihood	77.011	

estimated regime-switching function suggests that asymmetry stems from varying response of gas prices to oil prices during periods associated with (relatively big) gas price increases versus periods associated with price falls. Moreover, our results imply that such regime-wise asymmetry in the oil and gas price relationship was more pronounced before but not after the break.

4.3. Estimating the short run relationship

After having estimated the long run level relationship, we now turn to estimate an error correction model (ECM) to analyse short-run dynamics of oil-gas price relationship. Following the existing literature on gas and oil price relationship we also include the number of heating degree days (HDD), the deviation from the normal (prior 10-year average) number of heating degree days (HDDdev), the number of cooling degree days (CDD), the deviation from the normal (prior 10-year average) number of cooling degree days (CDDdev), deviation of the amount of natural gas storage from its average (Str), (first difference of

the natural logarithm of) the North American rotary rig count (rig) as control exogenous variables in the ECM (see, for example, Brigida, 2014).

After specifying an appropriate linear model,¹⁴ we carry out linearity tests. In linearity tests, in addition to lags of annual changes in oil and gas prices, we also consider the error correction term as possible regime switching variable. The results of linearity tests are reported in Table 6. As can be seen from the table, the null hypothesis of linearity is rejected for most of the candidate switching variables, implying that the short run dynamics also exhibit significant nonlinearities. Since the null of linearity is rejected more convincingly when the third lag of gas price

¹⁴ Maximum lag length of the changes in oil and gas prices were chosen by statistical significance of the last term included in the model as well as to eliminate significant residual autocorrelations. We also included two dummy variables for positive and negative outliers, which were defined as data points lying three standard errors away from the mean of the residuals.

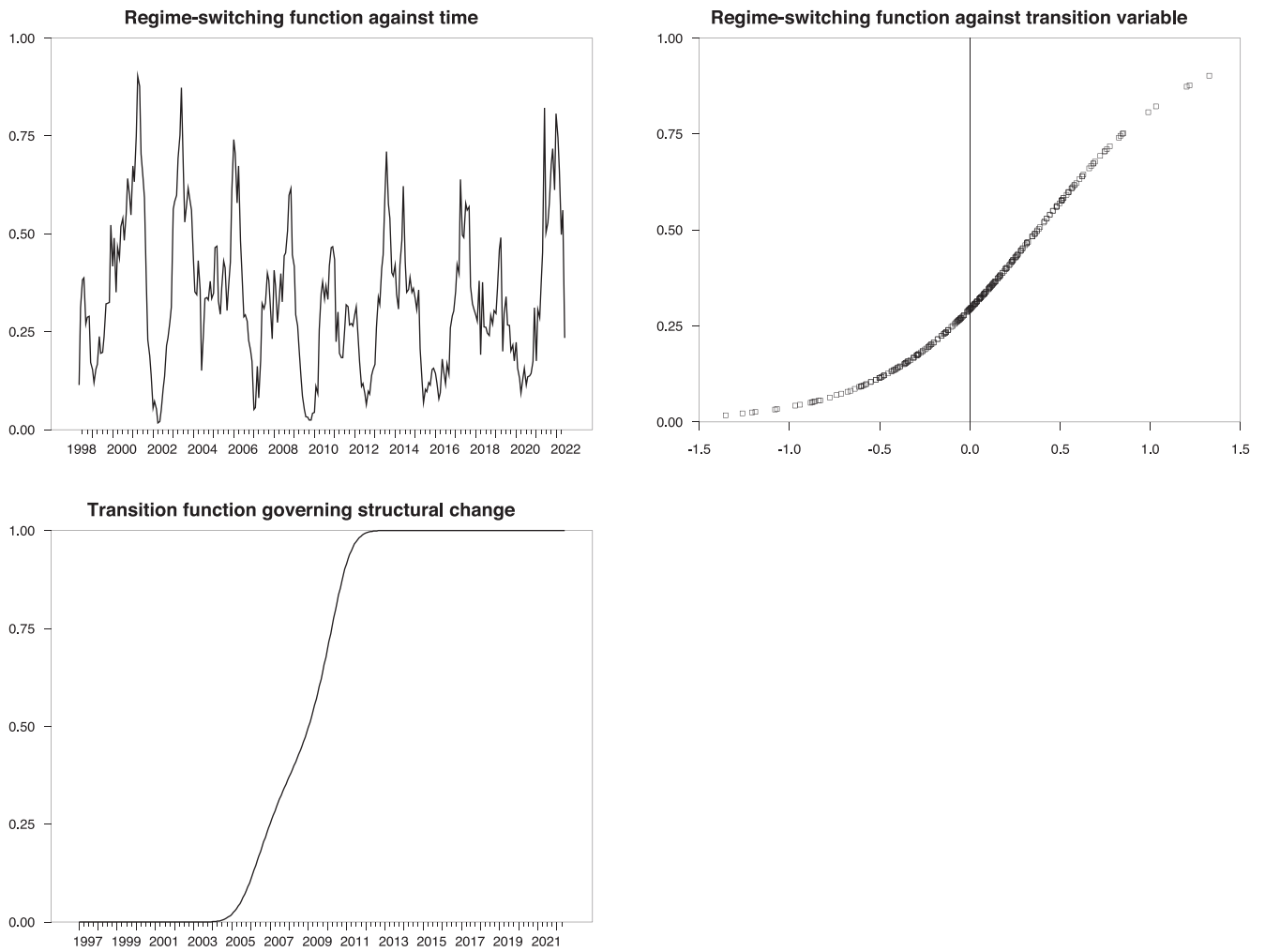


Fig. 3. Estimated transition functions for the long-run equilibrium relationship.

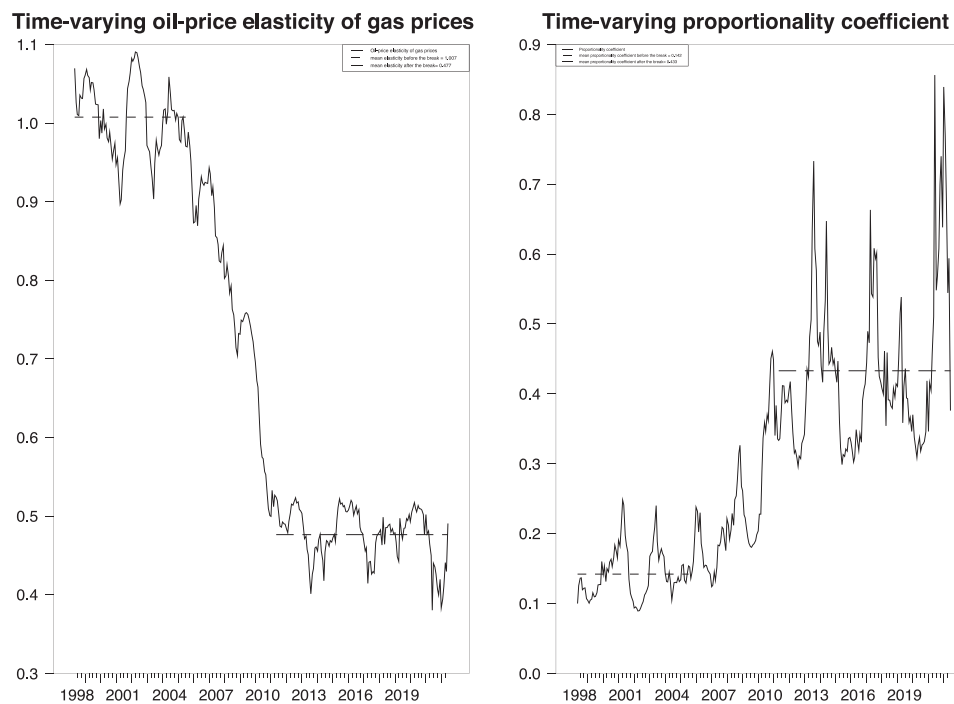


Fig. 4. Estimated time-varying coefficients of the long-run level relationship.

Table 6
Testing Linearity of the short-run relationship.

Candidate switching variable	Test statistic	p-value	Candidate switching variable	Test statistic	p-value
Panel A. Linearity tests against TV-STR type nonlinearity					
$\Delta_{12}op_{t-1}$	2.185***	1.01e-04	$\Delta_{12}gp_{t-1}$	2.275***	4.45e-05
$\Delta_{12}op_{t-2}$	2.105***	2.10e-04	$\Delta_{12}gp_{t-2}$	2.262***	5.04e-05
$\Delta_{12}op_{t-3}$	1.924***	0.001	$\Delta_{12}gp_{t-3}$	2.804***	3.02e-07
$\Delta_{12}op_{t-4}$	2.100***	2.24e-04	$\Delta_{12}gp_{t-4}$	2.479***	6.82e-06
$\Delta_{12}op_{t-5}$	2.372***	1.89e-05	$\Delta_{12}gp_{t-5}$	2.671***	1.12e-06
$\Delta_{12}op_{t-6}$	2.021***	4.58e-04	$\Delta_{12}gp_{t-6}$	2.603***	2.16e-06
$\Delta_{12}op_{t-7}$	1.727***	0.005	$\Delta_{12}gp_{t-7}$	2.407***	1.39e-05
$\Delta_{12}op_{t-8}$	1.473**	0.037	$\Delta_{12}gp_{t-8}$	2.041***	3.90e-04
$\Delta_{12}op_{t-9}$	1.294	0.117	$\Delta_{12}gp_{t-9}$	1.872***	0.002
$\Delta_{12}op_{t-10}$	1.544**	0.022	$\Delta_{12}gp_{t-10}$	1.846***	0.002
$\Delta_{12}op_{t-11}$	1.637**	0.011	$\Delta_{12}gp_{t-11}$	2.014***	0.001
$\Delta_{12}op_{t-12}$	1.456**	0.042	$\Delta_{12}gp_{t-12}$	1.795***	0.003
ect_{t-1}	1.800***	0.003			
Panel B. Transition function specification tests					
	Test statistic	p-value		Test statistic	p-value
H_{03}^{STR}	0.909	0.606	H_{03}^{TV}	1.141	0.295
H_{02}^{STR}	1.186	0.244	H_{02}^{TV}	1.389	0.098
H_{01}^{STR}	3.069***	1.12e-06	H_{01}^{TV}	1.975***	0.003

$\Delta_{12}X_t \equiv X_t - X_{t-12}$.

*** and ** denote rejection of the null hypothesis of linearity at 1% and 5% significance levels, respectively.

We use the F-variant of the linearity tests that provides better small-sample properties. See, for example, [Teräsvirta \(1994\)](#) and [Lundbergh et al. \(2003\)](#). Boldface denotes selected transition variable.

changes was used as a switching variable, we pick this variable as the regime-switching variable for modelling asymmetries in the short-run relationship. Notice that the H_{03}^{STR} and H_{02}^{STR} hypotheses are not rejected at conventional significance levels. Similarly, the H_{03}^{TV} hypothesis is not rejected whereas the H_{02}^{TV} hypothesis is marginally rejected at 10% significance level. On the other hand, the H_{01}^{TV} hypothesis is strongly rejected. Thus, model specification tests suggest that the first order logistic function is the most appropriate function to model both regime-wise nonlinearities as well as structural shift in the short-run relationship between crude oil and natural gas prices.

Having determined the most appropriate transition variable and transition functions, we estimate a time-varying logistic smooth transition error-correction model (ECM) for short run dynamics. Estimates of both linear and nonlinear ECMs are reported below in [Table 7](#). The graphs of the estimated transition functions governing regime switches and structural break are presented in [Fig. 5](#) below.

The estimated transition function that governs the structural shift suggests that the break was almost instantaneous around the end of the 2008.¹⁵ The estimated regime-switching function implies a threshold-type nonlinearity in the short-run dynamics of natural gas prices in which regimes are associated with falling and rising gas prices.

In order to present a better picture of structural break and

¹⁵ Estimated transition function suggests that the break began in August 2008 and was over by January 2009.

asymmetric effects in the short-run relationship, we plot graphs of the adjustment coefficient (coefficient on the error correction term) as well as the sum of the coefficients on oil price changes, which we refer to as the short-run oil price effect, in [Fig. 6](#).

First notice that the range of fluctuation of the adjustment coefficient has risen significantly after the break. In fact, while the adjustment coefficient varied within the interval $[-0.409, -0.332]$ before the break, the range of variation was $[-0.419, -0.229]$ after the break, implying almost negligible change in the regimes associated with falling gas prices. On the other hand, the adjustment coefficient for the periods associated with rising gas prices fell (in absolute value) moderately. However, a formal test of equality of adjustment coefficients suggest no change after the break. In particular we tested the null hypothesis that $H_0 : \gamma_l^b = \gamma_l^a$ and $\gamma_h^b = \gamma_h^a$ (where $\gamma_l^b, \gamma_l^a, \gamma_h^b, \gamma_h^a$ are the coefficients on the error correction term for the four different regimes associated with $\widehat{F}(\Delta_{12}gp_{t-3}) = 0 \cap \widehat{G}(\tau) = 0$ (i.e., lower regime before the break), $\widehat{F}(\Delta_{12}gp_{t-3}) = 1 \cap \widehat{G}(\tau) = 0$ (i.e., upper regime before the break), $\widehat{F}(\Delta_{12}gp_{t-3}) = 0 \cap \widehat{G}(\tau) = 1$ (i.e., lower regime after the break), and $\widehat{F}(\Delta_{12}gp_{t-3}) = 1 \cap \widehat{G}(\tau) = 1$ (i.e., upper regime after the break), respectively). The calculated test statistic was $\chi_2^2 = 2.299[0.317]$, suggesting no statistical evidence supporting change in the adjustment coefficient. We also tested whether there was regime-wise asymmetries in the adjustment coefficient. In particular, we tested $H_0 : \gamma_l^b = \gamma_h^b$ and $H_0 : \gamma_l^a = \gamma_h^a$, with associated test statistics $\chi_1^2 = 0.412[0.521]$ and $\chi_1^2 = 1.878[0.171]$, suggesting regime-wise asymmetries neither before nor after the break. These results imply that the mean of the adjustment coefficient was neither regime-dependent nor subject to structural breaks while the fluctuation of the adjustment coefficient might have widened significantly after the break.

The short-run impact of oil price changes on gas prices has fallen significantly after the break. While the short-run impact of oil price changes on gas prices was on average 0.759 ($Range = [0.563, 0.994]$) before the break, it has fallen to 0.122 ($Range = [0.097, 0.150]$) after the break. Formal test of equality of the sum of the coefficients also suggest significant change in the average oil price effects after the break. Specifically, the null hypothesis $H_0 : \sum \beta_{0lj}^b = \sum \beta_{0lj}^a$ and $\sum \beta_{0hj}^b = \sum \beta_{0hj}^a$, $j = 0, 1, 2, 3$, produced test statistic $\chi_2^2 = 9.237[0.010]$. On the other hand, we found no statistical evidence supporting regime-wise asymmetries in the oil price effects on gas prices. Both null hypotheses $H_0 : \sum \beta_{0lj}^b = \sum \beta_{0hj}^b$ and $H_0 : \sum \beta_{0lj}^a = \sum \beta_{0hj}^a, j = 0, 1, 2, 3$ were rejected with $\chi_1^2 = 0.075[0.411]$ and $\chi_1^2 = 0.042[0.838]$, correspondingly.¹⁶

These results provide a significant statistical evidence in favour of structural break and regime-wise asymmetries in the short-run gas price dynamics as well. In particular, the results suggest that the short-run response of the natural gas prices to oil prices has significantly fallen after the break. While a one percentage increase in oil prices caused gas prices to rise by 0.76% in the short run on average before 2008, the short run effect of oil prices has fallen to 0.12% after 2009. We also find significant structural shifts in and asymmetric response of natural gas prices to demand (climate-related variables) and supply conditions (storage and rig counts). Our results also imply a substantial increase in the volatility of the speed of adjustment of gas prices to deviations from the equilibrium relationship after 2009 while we find that that the

¹⁶ We also tested for regime-wise symmetry and structural break for the control variables. These tests, which are not reported here to save space, but are available upon request, suggest statistically significant structural change in coefficients of all control variables after the break. Also these results suggest regime-wise asymmetries in the effects of *str* variable both before and after the break, a regime-wise asymmetry in the effects of *HDDDev* variable before the break only, a regime-wise asymmetry in the effects of *CDDDev* and *rig* variables after the break only, and no regime-wise asymmetries in the effects of *HDD* and *CDD* variables.

Table 7
Estimates of the Linear and Nonlinear ECMs.

Panel A. Coefficient estimates of the linear error correction model					
Variable	Coefficient Estimate	Standard Error (p-value)	Variable	Coefficient Estimate	Standard Error (p-value)
Constant	0.030	0.021(0.169)	Str_t	-0.019	0.019(0.311)
Δect_{t-1}	-0.323	0.040(0.000)	HDD_t	-0.004	0.033(0.912)
Δgp_{t-1}	0.133	0.044(0.003)	$HDDdev_t$	0.104	0.012(0.000)
Δgp_{t-2}	0.093	0.044(0.036)	CDD_t	-0.235	0.086(0.007)
Δgp_{t-3}	0.064	0.043(0.137)	$CDDdev_t$	0.200	0.032(0.000)
Δgp_{t-4}	0.085	0.043(0.049)	rig_t	0.090	0.119(0.452)
Δop_t	0.193	0.055(0.001)	D_t^+	0.299	0.035(0.000)
Δop_{t-1}	-0.034	0.059(0.569)	D_t^-	-0.323	0.036(0.000)
Δop_{t-2}	0.196	0.061(0.001)			
Residual diagnostic tests:					
\bar{R}^2	0.606				
Log Likelihood	290.65				
SEE	0.091				
Sk	0.207	[0.155]	ARCH(12)	14.179	[0.289]
Ku	0.216	[0.460]	Q(12)	8.666	[0.731]
J-B test	2.606	[0.272]	B-P test (χ_{16}^2)	37.207	[0.002]
Panel B. Coefficient estimates of the non-linear error correction model					
Before the break			Upper regime: $F(\Delta_{12}gp_{t-3}) = 1$		
Lower regime: $F(\Delta_{12}gp_{t-3}) = 0$			$G(t) = [1 + \exp\{-\theta_2(t-c)\}]^{-1} = 0$		
Constant	0.141	0.038(0.000)	Constant	0.095	0.032(0.003)
Δect_{t-1}	-0.409	0.112(0.000)	Δect_{t-1}	-0.332	0.045(0.000)
Δgp_{t-1}	0.073	0.126(0.564)	Δgp_{t-1}	0.165	0.066(0.012)
Δgp_{t-2}	0.317	0.138(0.022)	Δgp_{t-2}	0.221	0.059(0.000)
Δgp_{t-3}	0.248	0.151(0.101)	Δgp_{t-3}	0.217	0.063(0.001)
Δgp_{t-4}	0.138	0.115(0.233)	Δgp_{t-4}	0.308	0.053(0.000)
Δop_t	0.421	0.170(0.013)	Δop_t	0.587	0.119(0.000)
Δop_{t-1}	0.216	0.109(0.048)	Δop_{t-1}	0.037	0.113(0.741)
Δop_{t-2}	0.357	0.171(0.036)	Δop_{t-2}	0.046	0.123(0.710)
Str_t	-0.273	0.105(0.009)	Str_t	-0.024	0.045(0.593)
HDD_t	-0.006	0.092(0.946)	HDD_t	-0.154	0.053(0.003)
$HDDdev_t$	0.023	0.056(0.672)	$HDDdev_t$	0.153	0.021(0.000)
CDD_t	-0.605	0.201(0.003)	CDD_t	-0.510	0.120(0.000)
$CDDdev_t$	0.130	0.134(0.329)	$CDDdev_t$	0.252	0.036(0.000)
$drig_t$	-1.159	0.384(0.003)	$drig_t$	-0.478	0.410(0.243)
After the break			Upper regime: $F(\Delta_{12}gp_{t-3}) = 1$		
Lower regime: $F(\Delta_{12}gp_{t-3}) = 0$			$G(t) = [1 + \exp\{-\theta_2(t-c)\}]^{-1} = 1$		
Constant	-0.051	0.046(0.265)	Constant	-0.034	0.035(0.324)
Δect_{t-1}	-0.229	0.073(0.002)	Δect_{t-1}	-0.417	0.116(0.000)
Δgp_{t-1}	-0.071	0.099(0.473)	Δgp_{t-1}	0.145	0.079(0.066)
Δgp_{t-2}	0.009	0.114(0.934)	Δgp_{t-2}	0.004	0.071(0.955)
Δgp_{t-3}	0.065	0.089(0.465)	Δgp_{t-3}	-0.071	0.056(0.204)
Δgp_{t-4}	0.003	0.084(0.972)	Δgp_{t-4}	0.027	0.049(0.582)
Δop_t	0.021	0.075(0.779)	Δop_t	0.180	0.122(0.141)
Δop_{t-1}	-0.035	0.071(0.623)	Δop_{t-1}	-0.307	0.106(0.004)
Δop_{t-2}	0.164	0.089(0.066)	Δop_{t-2}	0.224	0.151(0.138)
Str_t	0.079	0.029(0.006)	Str_t	-0.095	0.033(0.005)
HDD_t	0.063	0.070(0.373)	HDD_t	0.080	0.057(0.160)
$HDDdev_t$	0.144	0.024(0.000)	$HDDdev_t$	0.103	0.020(0.000)
CDD_t	0.013	0.179(0.944)	CDD_t	-0.078	0.118(0.508)
$CDDdev_t$	0.295	0.052(0.000)	$CDDdev_t$	0.132	0.034(0.000)
$drig_t$	0.033	0.187(0.862)	$drig_t$	1.522	0.268(0.000)
Estimates of the coefficients of the transition functions					
Coefficient estimates of the transition function governing structural change:					
$G(t) = [1 + \exp\{-\theta_2(t-\tau)\}]^{-1}$					
θ_2^*	533.21	382.72(0.164)	τ	0.464	0.002(0.000)
Coefficient estimates of the transition function governing structural change:					
$F(\Delta_{12}gp_{t-3}) = [1 + \exp\{-\theta_1(\Delta_{12}gp_{t-3}-c)\}]^{-1}$					
θ_1	236.33	302.53(0.435)	c	-0.054	0.009(0.000)
Coefficient estimates of the dummy variables:					
D_t^+	0.308	0.034(0.000)	D_t^-	-0.327	0.035(0.000)
Residual diagnostic tests:					
\bar{R}^2	0.692				
Log Likelihood	354.74				
SEE	0.080				
Excess skewness	0.091	[0.529]	ARCH(12)	15.153	[0.233]
Excess kurtosis	0.325	[0.266]	Q(12)	14.630	[0.262]
J-B test	1.667	[0.435]	B-P test (χ_{61}^2)	75.353	[0.102]

“Before the break” refers to the period prior to August 2008, and “After the break” refers to the period after January 2009. “Lower regime” refers to periods associated with falling gas prices (when fall in gas prices exceeds 6.6% p.a., i.e., $\Delta_{12}gp_{t-3} < -0.066$) and the “Upper regime” refers to periods associated with rising gas prices or a slight fall in prices (when change in gas prices is positive or not below -2% p.a., i.e., $\Delta_{12}gp_{t-3} > -0.02$). Storage is measured as Tcf (trillion cubic feet). HDD and CDD values are multiplied by a factor 0.001. HDDdev and CDDdev values are multiplied by a factor 0.01. ARCH(12) denotes Engle (1982) LM test against conditional heteroscedasticity. Q(12) denotes Ljung and Box (1978) Q-test for residual autocorrelation of order 12. B-P test is Breusch and Pagan (1979) test against heteroscedasticity. J-B is Jarque and Bera (1987) test for normality of residuals. p-values of residual diagnostic tests are shown in square brackets. Standard errors are heteroscedasticity consistent errors.

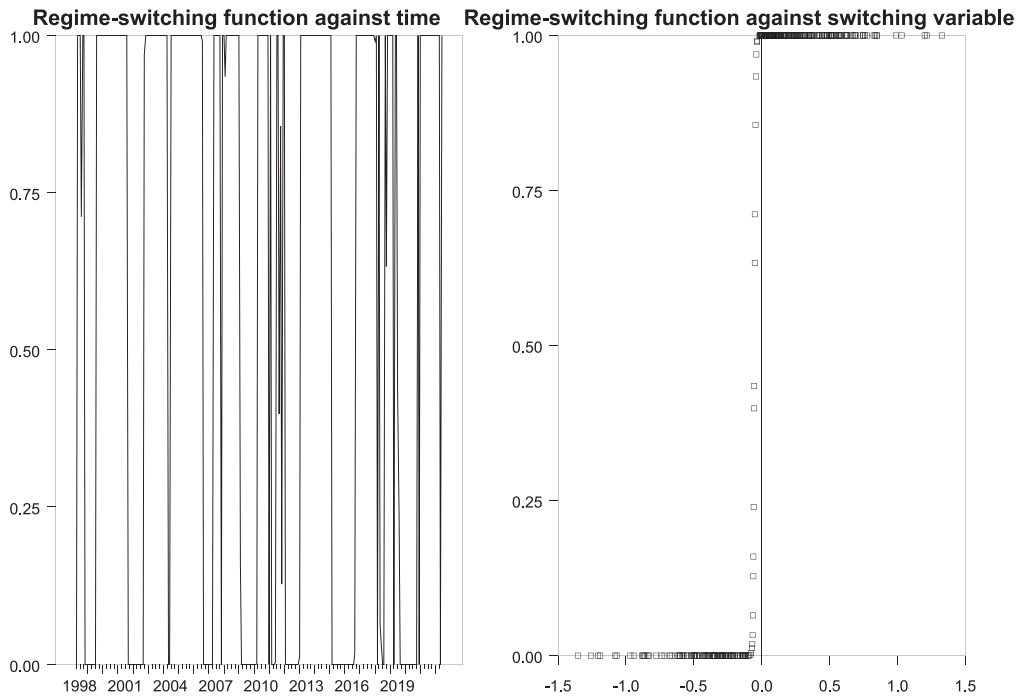


Fig. 5. Estimated transition functions for the short-run relationship.

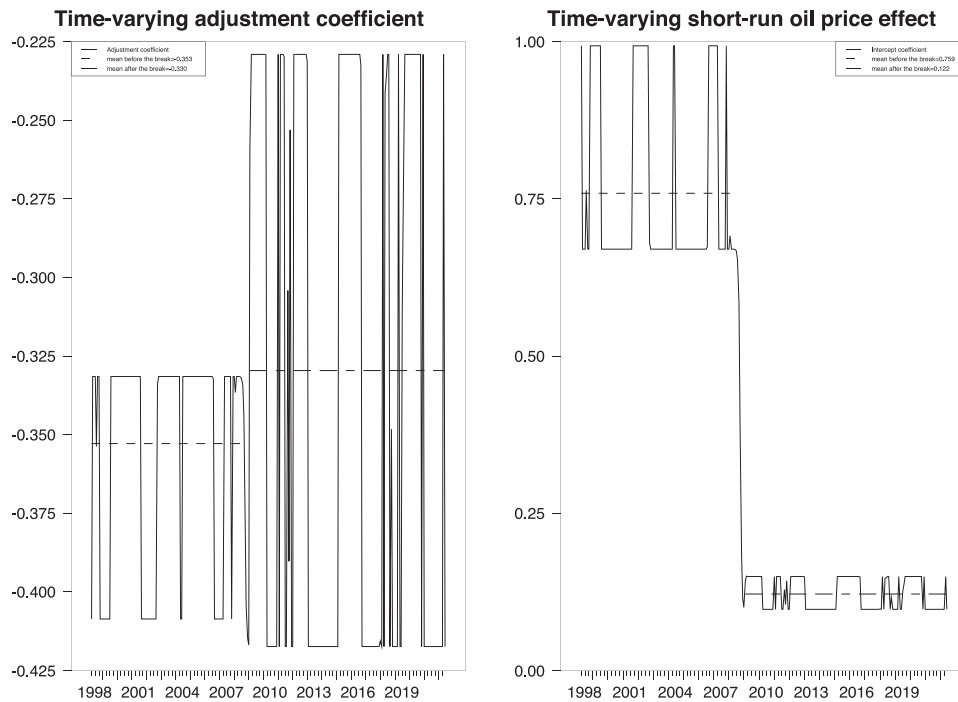


Fig. 6. Time-varying adjustment coefficient and short-run oil price effects.

average speed remained the same. Overall, our results suggest that the gas prices became less sensitive to oil prices both in the short and long runs and the relationship became more volatile after the break. The rise in the volatility of both the short- and long-run relationships may explain why some researchers (e.g. Erdős, 2012; Zhang and Ji, 2018) failed to find equilibrium relationship between the WTI and Henry Hub prices for the post 2008 period.

5. Discussion and policy implications

The results presented in the previous section provide new insights into the relationship between crude oil and natural gas prices. First, we find that the relationship between the prices is more complex than previously documented. In particular, our results imply that the relative oil and gas prices vary considerably across economic regimes and over time. Moreover, we find that the nonlinearities across regimes might also change over time. While previous researchers also provided evidence of structural breaks (e.g., Erdős, 2012; Chevallier and Ielpo, 2013; Caporin and Fontini, 2017; Zhang and Ji, 2018) and nonlinearities (e.g., Atil et al., 2014; Brigida, 2014; Asche et al., 2017; Batten et al., 2017), they analysed these features separately. Our results, on the other hand, imply that the oil and gas price relationship is more complex than previously thought and simple break or nonlinear models (such as threshold and Markov regime switching models) might not be appropriate to analyse the dynamic relationship between the prices. The failure to take account of either the nonlinearity or breaks will lead to model misspecification and hence produce biased estimates. This also explains why some researchers (e.g., Erdős, 2012; Zhang and Ji, 2018) failed to find cointegration between the crude oil and natural gas prices although they allowed for structural breaks.

Second, we find that the short-run relationships between the prices may also vary considerably over time and across regimes. In analysing either breaks or nonlinearities in the relationship between oil and gas prices, researchers have focused only on the long-run equilibrium relationship. Our results reveal that there are significant shifts and considerable nonlinearities in the short-run relationships even after controlling for breaks and nonlinearities in the equilibrium relationship. We also find that the short-run response of gas prices to supply and demand conditions also vary considerably both across regimes and over time, adding further complexity to gas price dynamics.

Third, we find that both the switches between the regimes and shifts across time was rather gradual in long run. Previous researchers who examined structural breaks (e.g., Erdős, 2012; Chevallier and Ielpo, 2013; Caporin and Fontini, 2017) assumed instantaneous change in the relative prices. Our results, on the other hand, imply that the shift in the relative prices was rather slow and took about six years to complete. Contrary to the long-run equilibrium relationship, we find that both the regime switches and shifts across time were rather sharp in the short run dynamics. Abrupt shifts in responses of natural gas prices to economic variables in the short run may be due to the fact that short-run dynamics are mainly determined by trading strategies. As traders may alter trading strategies without incurring significant costs, switches between regimes is likely to be instantaneous in the short run. However, changes in the long-run relationship require adjustment of the fundamental factors. In fact, adjusting either production or consumption of fuel cannot be done instantaneously, which in turn explains why both structural break and regime-switches in the equilibrium relationship take a longer period of time.

Our findings have clear implications for policy authorities and market practitioners as well. Yücel and Guo (1994) discussed relevance of the relationship between crude oil, natural gas and coal prices for designing optimal energy tax policies. They rightfully argued that optimal fuel tax policies eventually depended on the ultimate goal of the tax policies. If the aim is to raise revenue, fuels must be taxed in proportion to the long-run elasticities. However, if the aim is to alter the relative prices, and hence, the consumption of fuels, it is not necessary to tax all

fuels. Our results imply that the relative prices are highly nonlinear, and hence, there is no simple optimal tax rate. Also, as the relative prices are prone to big shifts over time, tax policy shall also be proactive. Furthermore, the fact that relative prices have varied significantly over time suggests that the so-called Lucas critique might be valid for energy policies as well: major policy changes affecting relative prices or supply and demand conditions will also cause to significant changes in consumption and production patterns of economic agents. Hence, using relationships observed in the past might not be appropriate in assessing effects of energy policies if these policies are expected to have significant effects on supply or demand conditions.

The results also imply that tax policies aimed at affecting fuel mix through changing relative prices will have little effect as changes in prices of one of the fuels will be reflected in the price of the substitute fuel as well. Furthermore, the long-run oil price elasticity of natural gas prices have halved post 2011 period, implying a significant change in substitutability between fuels. However, as the gas prices did not decouple from oil prices completely, there is still some room for substitution, albeit as not much as before. Asche et al. (2006) argued that cointegration of prices imply integration of energy markets, and hence, the existence of a single market for energy. If this is the case, energy consumers do not put any emphasis on the carrier, only on the energy it provides. Hence, these results also imply that direct subsidies might be more effective in promoting clean energy consumption than indirect taxes on fossil fuels. Overall, energy tax policies must take account of the complex nonlinear and time-varying nature of relationship between energy variables.

Our results are valuable for energy market participants as well. The existence of the long-run relationship implies that both consumers and investors can hedge against adverse price changes as well excessive price volatilities in the gas market using oil derivatives (see also Chevallier and Ielpo, 2013; Atil et al., 2014). However, our results suggest that hedging is sensible only for the long-term risks. In fact, as the speed of adjustment of gas prices towards the equilibrium level is low and highly volatile, engaging in the short run hedging strategies might be undesirable. Furthermore, the fall in the short-run sensitivity of gas prices to oil prices also undermines usefulness of oil prices as a hedging tool against gas price volatility in the short run. Thus, our results imply that the relative fuel prices could effectively be used for hedging only against the long-term risks whereas there is little room, if any, for hedging short-term adverse price risks using relative price dynamics. By the same token, our results also suggest little room for profitable trading strategies in energy markets. In fact, high volatility of price responses to deviations from the equilibrium as well as reduced sensitivity of gas prices to oil prices dilute profitability of any trading strategies that are based on past price relationships.

Although our results point to possibility of using relative prices to hedge against adverse price risks, the results reported in this study are not sufficient to develop a profitable hedging strategy as we focus only on the relationship between the first moments (i.e., mean equations) but ignore the structure of the second moments (i.e., variance equations). In fact, developing profitable hedging strategies requires a good understanding of the relationship between both the first and second moments of the related variables. Also, it is well documented that variances of crude oil and gas prices are highly correlated (e.g., Pindyck, 2004; Lin and Li, 2015). As we used monthly data, we found no heteroscedasticity in the residuals. However, high frequency data (e.g., daily and weekly) are more likely to exhibit time-varying variance. Furthermore, the relationship between high-frequency data is more useful for practical hedging decisions. Therefore, further empirical analysis on the relationship between the first and second moments of oil and gas price series is needed for building practical hedging strategies. But such analysis must allow for simultaneous structural breaks and nonlinearities as well because misspecification of the mean equation will lead to misspecification of the variance equation as well.

Dynamics of relative fuel prices are crucial for energy producers and

consumers as optimization (i.e., cost minimization and profit maximization) decisions depend on relative prices. Relative fuel prices are the main determinant of the fuel demand. Fuel prices are important for producers as well. In fact, major natural gas producers also produce crude oil and oil products. Gas and oil producers compete for the same factors of production. In addition, associated gas (i.e., natural gas produced as a by-product of crude oil extraction) accounts for a significant share of gas production whereas reinjection of gas might be required to sustain productivity of oil wells. Therefore, relative crude oil and natural gas prices are central to production decisions. The existence of a long-run equilibrium relationship between the oil and gas prices imply that producers need not to adjust optimal production decisions in the face of deviations of the relative prices from the equilibrium level as these deviations are not persistent and will fade with lapse of time. Similarly, consumers need not change the optimal energy mix in the face of temporary deviations from the equilibrium. However, as the long-run equilibrium relationship is prone to significant shifts over time, adopting more flexible consumption and production strategies that allow relatively low-cost changes in energy mix or production plans might be more cost effective in the long run.

Many long-term gas contracts use crude oil prices as a benchmark. While the existence of the long-run equilibrium relationship between oil and gas prices can be seen as an empirical rationale for such benchmarking, significant shifts in the relationship questions rationality of using past observed relationship for the future pricing mechanisms. Furthermore, we find strong evidence of asymmetry in the dynamics of gas prices both in the short and long run, which further dilute rationality of oil indexation. In addition, Zhang et al. (2018) argued that oil indexation may produce more abnormal price movements, i.e. “bubbles” in gas markets. In fact, price bubbles in international oil markets are transmitted to gas prices through indexation, and usually with a lag. As “the pricing system ignores fundamental values in natural gas, the market tends to react to information irrationally and thus creates more bubbles than are passed from the oil market”. Therefore, given highly nonlinear and time-varying nature of the relative fuel prices, oil indexation may lead to more price volatility in gas prices.

All in all, our results imply that both the long- and short-run relationship between crude oil and natural gas prices are highly nonlinear. Both researchers and market practitioners must consider the possibility of structural shifts and nonlinearities in analysing energy data.

6. Conclusion

In this paper we re-examined the natural gas and crude oil price relationship by allowing for possible structural shifts and nonlinearities. For this purpose, we adopted a more flexible smooth transition regression (STR) modelling approach that allows modelling simultaneous structural breaks and regime-wise nonlinearities. A nice feature of the STR modelling approach is that it does not impose a functional form and allows to decide based on specification tests whether a full time-varying and regime-switching model is necessary to analyse the relationship between variables or a nested sub-models such as a time-varying model or a regime-switching model is sufficient to describe the relationship between the variables. Also, this modelling approach allows one to choose the most appropriate forms of the regime-switching functions that depict nonlinearities and structural shifts.

Our results suggest that the relationship between crude oil and natural gas prices are more complex than previously documented. Specifically, we find strong and robust statistical evidence in favour of simultaneous structural break and nonlinearities both in the long- and short -run relationship between crude oil and natural gas prices. Our estimates imply that the equilibrium relationship between gas prices and oil prices have shifted significantly during the analysed period. Specifically, we find that gas prices became less sensitive to oil prices both in the short and long run after the break. We also find that the structural shift in the long run equilibrium relationship was rather slow. The

results also imply that gas prices did not decouple from the oil prices permanently but the equilibrium relationship moved to a new level.

These results have clear and nice implications for both researchers and practitioners. First, one must take account of both structural breaks and nonlinearities in analysing energy data. Second, observed relationship between the crude oil and gas prices in the past might not be guiding for the future relationship, especially in the face of significant supply and demand shocks. Also, adopting more flexible consumption strategies that allow easy switch between fuels might be more cost effective in the long run. Finally, our results also suggest that oil prices could be used to hedge against risks associated with large swings in gas prices, but only against the long term risks.

Our study have some limitations as well. We focused only on the relationship between mean of the price series. However, it is well documented that variances of crude oil and gas price series are also highly correlated. Practical application of the results of this paper for hedging strategies require a good understanding of dynamics of variance as well. Most importantly, all practical decisions by governments, producers and consumers as well as investors and traders in the energy market in real world are made under uncertainty, which, in addition to the relationship between mean of the series also require a good understanding of the dynamic relationships between variances of the price series. Further research is needed to examine possible nonlinearities and shifts in the variances of the oil and gas prices. The results of this paper also sheds light on modelling strategies for the future research on variances as misspecification of the mean equation will lead to misspecification of the variance equation as well. Also, detailed empirical analysis of structural shifts and nonlinearities in other regional markets as well as with high frequency data would add to our knowledge of the crude oil and gas price relationships.

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CRedit authorship contribution statement

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Declaration of competing interest

None.

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Appendix A. Supplementary data

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