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Application of Interval Valued Intuitionistic Fuzzy Uncertain MCDM Methodology for Ph.D Supervisor Selection Problem

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ABSTRACT

The selection of Ph.D (Doctor of Philosophy) supervisor is always a vital and interesting problem in academia and especially for students who want to carry out Ph.D. Nowadays, selecting a supervisor for Ph.D in a scientific manner becomes a challenge for any student because of the variety of options available to the scholar. In this context, the present study aims to formulate a model for Ph.D. supervisor selection from the offered alternatives in an academic institute. A hybrid multi-criteria decision making (MCDM) framework has been applied to select the suitable supervisor of the student's preferred criteria under interval-valued intuitionistic fuzzy (IVIF) scenario. The IVIF Analytic Hierarchy Process (AHP) has been employed to prioritize the criteria, whereas IVIF Technique for order preference by similarity to ideal solution (TOPSIS) technique is engaged to rank the available supervisors based on criteria weight. A set of eight criteria and five alternatives have been considered for modeling the problem. Moreover, the potential criteria are weighted and ranked by the multiple decision makers in the present study. To examine the consistency and robustness of the proposed integrated approach, sensitivity analysis and comparative analysis have been carried out. From all the analyses, it can be conferred that the suggested approach is quite useful to apply in different decision-making scenarios.

1. Introduction

Selection of a Ph.D (Doctor of Philosophy) supervisor is a significant step that should be taken at the beginning phase in the student research profession, and this is quite possibly the main component impacting on effective consummation of educational courses. When a specific Ph.D advisor is going to be selected, everyone must confirm every aspect of the supervisor to determine whether he/she is helpful or not. Phillips, E.M., et al. [1] suggest looking for positive answers to at least some of the questions such as, "Have they published research articles recently or not? Do they achieve any research grants or contracts? Are they invited to participate in conferences in abroad institutes?" [2]. Students could likewise be keen on being familiar with the nature of journals and conferences where the professors regularly publish, their collaborators, current and past student records, and so on.

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Table 1
Literature on relevant Ph.D. Supervisor selection problems with their details.

Author(s)	Year	Factor & sub- factors number	Environment	Optimization
[3] Ray, S.	2007	10 Factors	Fuzzy	AHP
[4] Ives, G. et al.	2007	11 Factors	Crisp	Not applicable
[5] Datta, S. et al.	2009	16 Factors	Fuzzy	COPRAS-G
[6] Momeni, M. et al.	2011	12 Factors	Fuzzy	ANP
[7] Barkovic, D. et al.	2014	6 Factors	Fuzzy	AHP
[2] Hasan, M.A. et al.	2019	4 Factors & 14 Sub-factors	Fuzzy	Fuzzy AHP, TF/IDF Algorithm
[8] Van Rooij, E. et al.	2019	18 Factors	Crisp	Multiple regression analysis
[9] Jabre, L. et al.	2021	10 Factors	Crisp	Not applicable
[10] Cardilini, A. P. et al.	2022	16 Factors	Crisp	Relative averaged rank
This study	2024	8 Factors	Fuzzy	AHP & TOPSIS

Consequently, students might well exploit a decision-making system that distinguishes the significant criteria and guides them in assessing supervisors concerning those criteria.

Table 1 describes the list of Ph.D supervisor selection problems in recent times with author name, number of factors and sub-factors, type environment and optimization techniques taken.

So, from the above table, we see that there is some recent work already done for supervisor selection in uncertain environments. That means a huge scope is here to extend the previous work.

At the very beginning of the work, we conducted a field survey among the 100 number of Ph.D completed persons and asked them few questions. The questions we ask from them are as follows:

(a) Is the research topic important for Ph.D supervisor selection?
(b) Are Past scholars and alumni status should be important factors?
(c) Is the collaboration of supervisors an important factor?
(d) Is regularly meeting with the supervisor required?
(e) Is the support by the supervisor for student mental health important?
(f) Are publications with due time followed by a supervisor guide important?

We give three options for answering the above question as (i). ‘Yes’, (ii). ‘No’ & (iii). ‘Can not say’. The output of the questionnaires is as follows:

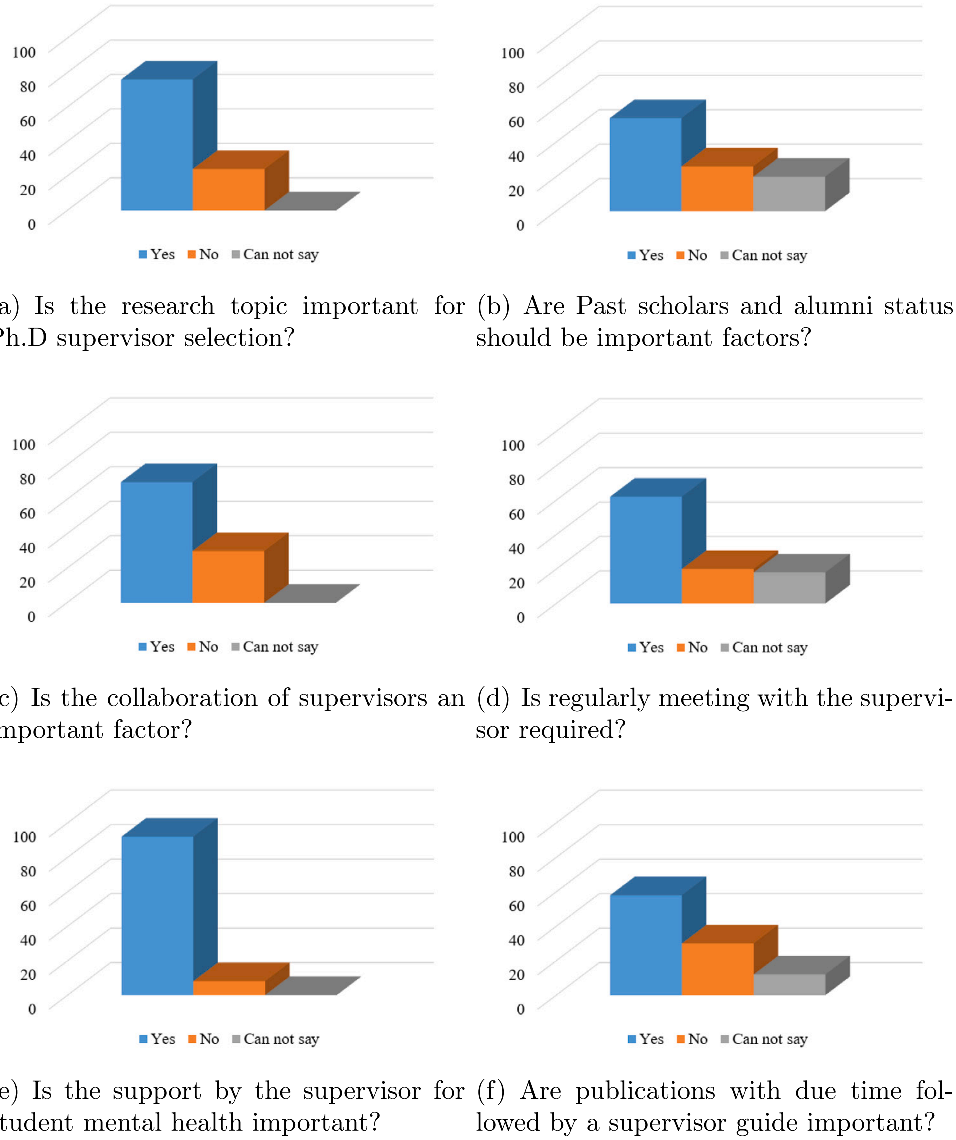
Fig. 1 depicts the survey results of six questions on Ph.D supervisor selection. Considering the above facts we are very much motivated how can we find a strategy for Ph.D supervisors concerning different factors and criteria. These six questions and their answers are given a strategy for the criteria selection process [9] of this study.

1.1. Motivation of this study

The direction of the supervisor is a significant determiner [6] of value in a doctoral thesis and subsequently plays a very important role for a scholar in his/her research achievement. Earning a doctorate degree, one of the highest academic degrees is a significant educational, professional, and personal achievement that takes years of hard work, determination and dedication [11]. We select the supervisor to evaluate the positive and negative characteristics of the alternatives. Sometimes, students fail to complete their courses due to proper guidance from the supervisor to complete the work. The selection of a Ph.D supervisor [8,10] is very complex because it improves the personal relationship between advisor and students as well, as the future profession of a scholar may be guided by the supervisor. Additionally, the research topic has a crucial component that varies depending on the supervisor. A researcher may have a preference for a research topic that they find particularly tough, as reported by their supervisor. Therefore, selecting a proper guide is a useful study and may be modeled as an MCDM problem [12] due to the presence of various criteria [4,9] as well as alternatives from which a student can choose a suitable guide who can help to complete his research in a fruitful direction to get a doctoral degree.

1.2. Novelties of the research

Some researchers applied the Analytic Hierarchy Process (AHP) technique [12] and Analytic Network Process (ANP) technique [13] for calculate the weight of the factors. Similarly, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method [14], Complex proportional Assessment (COPRAS) methodology [15] and some fundamental techniques are used to prioritize the alternatives. Two MCDM methodologies with an ordinary fuzzy number to select the most suitable Ph.D supervisor. However, many times, we may not measure the fuzzy data to get exactness with ordinary fuzzy numbers and membership functions but the interval valued fuzzy numbers [15] give perspicuity of fuzzy data. Moreover, the intuitionistic fuzzy number (IFN) [16], an extension of the fuzzy number [17], which contains both membership degree (μ) and non-membership degree (ν), can express the uncertainty better than the fuzzy set. So, in this study with the aid of AHP-TOPSIS methodologies, we have implemented an MCDM framework [12] in interval valued intuitionistic fuzzy (IVIF) environment [18] to choose a suitable supervisor. Here, each fuzzy data is presented by an interval-valued number to get a clear representation of uncertainty. The IVIF AHP method [19] will be used to prioritize criteria, and the IVIF TOPSIS technique [20] will be used to rank supervisors based on criteria weight.



(a) Is the research topic important for Ph.D supervisor selection?

(b) Are Past scholars and alumni status should be important factors?

(c) Is the collaboration of supervisors an important factor?

(d) Is regularly meeting with the supervisor required?

(e) Is the support by the supervisor for student mental health important?

(f) Are publications with due time followed by a supervisor guide important?

Fig. 1. Answers of six questions asked on Ph.D supervisor selection to Ph.D completed person.

1.3. *Objective and contribution*

We inquire how a student chooses a doctoral thesis supervisor from among all of the professors based on specific criteria. We take the paper [2,3,5–7,21] and try to study the criteria and sub-criteria taking in the problem, which is shown in Table 2. If anyone sees the criterion and sub-criterion, then it is very confusing for fresher students when they select a supervisor. For that reason, we take a few of the important criteria only, which are already discussed in Section 5. The supervisors are ranked by interval-valued intuitionistic fuzzy numbers (IVIFN) using the de-fuzzification of IVIFNs. The supervisors are ranked using data from a student who is seeking a Ph.D as well as combined data from three other persons from another department & institute.

1.4. *Structure of this research*

The introduction of this study is covered in Section 1. Literature surveys on this topic with different perspectives are discussed in Section 2. In Section 3 describe the basic concept of fuzzy set (FS) and its extension up to interval-valued intuitionistic fuzzy set (IVIFS). Also, discussed various properties and de-fuzzification methods of IVIFNs. Section 4 discoursed the AHP and TOPSIS based MCDM methodologies under IVIFN environment integrated with the supervisor selection model. Section 5 covered the research

Table 2
Different criteria and sub-criteria taken in different past studies.

Author(s)		Year	Criteria & sub-criteria	
[2]	Hasan et al.	2019	(*i*) Research area Relevance (*a*) Broad research interests (*b*) Taught courses research interests (*c*) Specific research interests/topics (*d*) Student's publication & dissertation record interest (*ii*) Publication Record (*a*) Publication record of professor's graduated student (*b*) consistency in publishing (*d*) Publication quality (*iii*) Collaboration Record (*a*) Research grant quality (*c*) Consistency in getting grant (*iv*) Research Grant Record (*a*) Recent collaboration record in research paper (*c*) Record as Co-PI/CO-I in research grant	(*c*) Recent publication record (*b*) Recent grant record (*b*) Influential Co-authors
[3]	Ray et al.	2007	(*i*) Reputation, publications (RP) (*iii*) Number of thesis guided (NT) (*v*) Personal relationship with the professor (PR) (*vii*) Convergence of interest (CI) (*ix*) Commitment and involvement (IN)	(*ii*) Time conscious (TC) (*iv*) Social networks (SN) (*vi*) Job prospect (JP) (*viii*) Can take a stand (CS) (*x*) Freedom to work (FW)
[5]	Datta et al.	2009	(*i*) Past record on research guidance (*iii*) Projects and consultancy (*v*) Friendly interaction with students (*vii*) Contacts in academic fraternity (*ix*) Extent of academic exploitation (*xi*) Reputation among the students whom previously guided (*xii*) Reputation among colleagues in the department (*xiii*) Knowledge in computer programming language (*xiv*) Dedication (Punctuality, involvement, extent to work hard) (*xv*) Depth of knowledge in his/her own field (*xvi*) Administrative position at the institute	(*ii*) Research publications (*iv*) Pedagogy of teaching (*vi*) Problem solving capacity (*viii*) Communication skill (*x*) Attitude like a "boss"
[6]	Momeni et al.	2011	(*i*) Convergence of Political Tendency (*iii*) Social Network for conduct the research (*v*) Social Network for getting job/opportunities (*vii*) Relationship with other faculty members (*ix*) Personal Characteristics (*xi*) Convergence of Interests	(*ii*) Scientific Reputation (*iv*) Commitment (*vi*) Knowledge (*viii*) Flexibility (*x*) Being Free (*xii*) Can take a stand
[7]	Barkovic et al.	2014	(*i*) Freedom to work (FW) (*iii*) Social networks (SN) (*v*) Personal relationship with the professor (PR)	(*ii*) Reputation, publications (RP) (*iv*) Number of thesis guided (NT) (*vi*) Time conscious (TC)
[21]	Jabre et al.	2021	(*i*) Align research interests (*iii*) Be sure to meet current students (*v*) Work style compatibility (*vii*) Consider the entire experience (*ix*) But also try to meet past students	(*ii*) Seek trusted sources (*iv*) It takes two to tango (*vi*) Trust your gut (*viii*) Expectations (*x*) Wash, rinse, repeat

guidance in academic and research institutes which are considered as criteria. Section 6 demonstrates various publication databases and other websites where we can learn about PhD supervisor's work records. The data sources and numerical results are illustrated in Section 7 in detail. Also, Section 8 and Section 9 investigated the sensitivity and comparative analysis briefly, respectively. Finally, the conclusion of this study, limitations of this study and some future research scope are described in Section 10.

2. Literature survey

Operations research has a section called multi-criteria decision making (MCDM) [22–24], when there are conflicting criteria to select an alternative in most real-world scenarios. The MCDM methodologies can be taken into consideration to make decisions considering numerous criteria and sub-criteria from various options. Fuzzy sets [25] and fuzzy decision-making [26] methods are able to effectively manage the ambiguity and imprecision that are often inherent in decision-making. The theoretical and practical elements of MCDM and fuzzy MCDM have been the subject of considerable research areas in recent eras. There are several methods to find the significance of the criteria in choosing an alternative. AHP, TOPSIS, COPRAS, Weighted Aggregated Sum Product Assessment (WASPAS) [27], Multi-Objective Optimization on the basis of Ratio Analysis (MOORA) [28], MULTI-MOORA [29], and others are widely used MCDM techniques in real-life problems.

2.1. Literature review on IVIFS

K.T. Atanasov [30] presented the intriguing and practical IFS (Intuitionistic Fuzzy Set) theory for problem solving. In actuality, the fuzzy set defined by Zadeh [31] is extended by the IFS. An IVIFN or IVIFS is a mathematical construct that combines fuzzy logic and intuitionistic logic while allowing for uncertainty in the membership, non-membership, and hesitation degrees by representing them as intervals. This makes IVIFNs a powerful tool in decision-making processes where uncertainty and imprecision are inherent, providing a more flexible and realistic presentation of vague information. The fuzzy set concept is one of the innovative inventions invented by Prof. Lotfi A. Zadeh [31] in 1965. There are so many extensions of fuzzy sets designed by various studies. The intuitionistic fuzzy set [30], interval valued fuzzy set [32], Pythagorean fuzzy set [33], spherical fuzzy set [34], triangular fuzzy set (TFS) [35], trapezoidal fuzzy set (TrFS) [36], pentagonal fuzzy set (PFS) [37] and hexagonal fuzzy set (HFS) [27] are the extensions of the fuzzy set. The interval valued intuitionistic fuzzy set (IVIFS) [38] is another extensional approach of the fuzzy set. This study considers the IVIFS as an uncertainty variable.

There are several applications of interval valued intuitionistic fuzzy set (IVIFS) in recent eras, some are discussed here. Wan, S. et al. [39] apply the possibility degree methodology for selection of air-condition system and analysis using IVIFS, Wan, S. et al. [40] used the interval-valued intuitionistic fuzzy (IVIF) multi criteria group decision making (MAGDM) to reduce the risk and increase profits by a venture capital company. The IVIF truth degrees applied by Wan, S. P. et al. [41] in heterogeneous multiattribute decision-making techniques to utilize the Green supply selection problems and Wan, S. et al. [42] in hybrid multi-criteria group decision making (GDM) methods to check the performance of critical infrastructure. Aso, Wan, S. et al. [43,44] applied IVIF in GDM to solve the selection problem for enterprise partner and evaluation of information technology (IT) outsourcing service providers, respectively. Wan, S. et al. [45,46] applied IVIFN to evaluate the efficiency of enterprise resource planning (ERP) and selection of network systems, respectively using multi-criteria GDM methodology. The IVIFS is also used in the selection of virtual enterprise partners [47], the automotive component's material [48] and risk assessment among the COVID-19 prohibition [49]. The effectiveness & validation of the product is determined by Best-worst method (BWM) based MCDM method by Dong, J. et al. [50] and superiority & simulation analysis of the outcome is evaluated by MCDM techniques by Dong, J. et al. [51] using IVIFN.

2.2. Literature review on different MCDM problems

The MCDM problem is a very important tool for making a decision for a complex real-life problem. The methods are not quite tough and also not so laborious. Paul, K. et al. [52] applied optimization techniques in residential buildings for energy optimization and Paul, K. et al. [53] applied MCDM methodology to battery energy and renewable energy efficiency measures. Optimization techniques are also used in reducing transmission network power system congestion [54], wind energy source on the transmission network of the power system using congestion management [55] and network system of power using Bat algorithm [56]. We are trying to study a few papers where the authors apply different MCDM techniques in several areas. Table 3 explains the different selection problems and detailed specifications of those studies.

Analytical Hierarchy Process (AHP) is a widely used decision-making methodology developed by Thomas L. Saaty [73] in 2008. It is particularly popular in fields such as management science, operations research, and engineering. Kang et al. [74] proposed an Analytic Hierarchy Process (AHP) and ArcGIS Multi-Criteria Decision-Making (MCDM)-based optimal landfill site selection for Kinshasa City. Shaikh et al. [75] introduced flood hazard mapping by linking CF, AHP, and fuzzy logic techniques in Urban Areas. There is also wide range of recent applications used by AHP in disaster management [76], company performance [77], solar farm site selection [70], plant site selection problem [71] and so on. The AHP method is also used in different recent problems which are described in Table 4.

The TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [57,59] is a multi-criteria decision-making (MCDM) methodology developed to accommodate the decision-makers in selecting the optimal alternative from possible alternatives based on multiple criteria. The TOPSIS technique was first invented by Hwang & Yoon [91] in 1981. At present time, this approach has grown in popularity across a range of industries thanks to its ease of use and efficiency in solving challenging decision-making issues. In the most recent applications, there has been a lot of work such as transportation [92], developing guiding principles [93], employee performance evaluation [57], teachers selection [59], corridor selection [18] and so on. Various recent studies on the TOPSIS method with real life applications are discussed in Table 5.

The IVIF AHP (Interval Valued Intuitionistic Fuzzy Analytical Hierarchy Process) method and IVIF TOPSIS (Interval Valued Intuitionistic Fuzzy Technique for Order of Preference by Similarity to Ideal Solution) technique are extensions of the traditional AHP and TOPSIS, respectively, that incorporate IVIFNs (Interval Valued Intuitionistic Fuzzy Numbers). These extensions are designed to handle uncertainty and vagueness in decision-making processes. Table 6 expresses the recent applications of IVIF AHP & IVIF TOPSIS methods in different papers.

3. Preliminaries of mathematical tools

This section discussed essential definitions and properties of fuzzy sets (FS), membership functions (MF), fuzzy numbers (FN), Triangular fuzzy numbers (TFN), intuitionistic fuzzy numbers (IFN) and interval valued intuitionistic fuzzy numbers (IVIFN). The de-fuzzification method of IVIFNs is proposed in this section.

Table 3
Literature survey on various site selection problems with their associated data.

Citation & Author(s)		Factor & sub-factors number	Environment	MCDM approach	Application area
[11]	Aghdaie et al.	9 Factors	Fuzzy environment	Fuzzy AHP, Fuzzy TOPSIS	Thesis subject selection
[57]	Rahmati et al.	8 Factors	Fuzzy environment	Fzzy AHP, Fuzzy TOPSIS	Employee performance evaluation
[58]	Özkan, T.K.	5 Factors	Fuzzy environment	ELECTRE	A school selection problem
[59]	Moayeri et al.	3 Factors & 16 Sub-factors	Fuzzy environment	Fuzzy AHP, Fuzzy TOPSIS	Math teachers selection
[60]	Ramlan, N.D.B.	5 Factors & 15 Sub-factors	Fuzzy environment	AHP, ELECTRE-I	Academic staff selection of faculty of science at educational institute
[61]	Kundakcı, N.	9 Factors	Fuzzy environment	AHP, OCRA	Tablet computer selection
[62]	Mahmoodzadeh, S. et al.	4 Factors	Fuzzy environment	Fuzzy AHP, TOPSIS	Project selection problem
[63]	Ertuğrul et al.	5 Factors	Fuzzy environment	Fuzzy TOPSIS	Academic member selection in engineering faculty
[64]	Biswas et al.	8 Factors	Fuzzy environment	ENTROPY, Modified SAW	Selection of an IIT
[29]	Miç, P. et al.	5 Factors	Crisp environment	TOPSIS, WASPAS, MULTIMOORA	University location selection
[65]	Baser, V.	4 Factors & 9 Sub-factors	Crisp environment	AHP	School site selection
[66]	Moussa, M. et al.	4 Factors	Crisp environment	Analytical methodology	Site selection of school
[67]	Biswas, S. et al.	12 Factors	Crisp environment	EOGDM	Location selection for B-schools
[68]	Shaikh, S.A. et al.	4 Factors	Crisp environment	AHP & TOPSIS	Ideal business location identification
[69]	Baser, V.	4 Factors & 9 Sub-factors	Crisp environment	AHP	School site selection
[70]	Wiguna, K.A. et al.	3 Factors & 6 Sub-factors	Fuzzy environment	FAHP & FPROMETHEE	Solar farm site selection
[22]	Mostafa, A.M.	9 Factors	Crisp environment	BOM, BWM & AHP	Cloud computing service selection
[71]	Kaboli, A. et al.	5 Factors	Fuzzy environment	FAHP	Plant location selection
[72]	M.H.V. et al.	5 Factors	Fuzzy environment	FAHP & $\alpha-$cut method	Hospital site selection
[23]	Nuriyev, A.M.	6 Factors	Fuzzy environment	Z-TOPSIS & Z-PROMETHEE	Selection of the tourism development site

3.1. *Fuzzy set and its properties*

Fuzzy sets were presented by Prof. Lotfi A. Zadeh [31] in 1965. The membership function (MF) is a characteristic that defines a fuzzy set, unlike a crisp set. The expressive, observational and personalized viewpoints are included in the fuzzy set. Linguistic variables are interfaced subjectively by linguistic terms and assessable by the fuzzy set in the universal set of discourse and represented by MF [125,126]. To reduce complexity, the fuzzy set eliminates the sharp boundary that separates the pair's members from non-members. Each element in a set along with its membership values [127]. The fuzzy set and membership functions are defined in detail as follows:

Definition 1 (*Fuzzy Set*)**.** Consider fuzzy set $\tilde{A}_{FS}$ in universal set of discourse X is defined as

$$\tilde{A}_{FS} = \left\{ \left(x, \mu_{\tilde{A}_{FS}}(x) \right); x \in X \right\} \tag{1}$$

in which $\mu_{\tilde{A}_{FS}}(x) : X \to [0, 1]$ is the membership function (MF) of $\tilde{A}_{FS}$.

The fuzzy set is written in an order pair where the first entry is the element itself and the second one is its membership value which always lies in $[0, 1]$. The membership values (MV) $\left(\mu_{\tilde{A}_{FS}}(y)\right)$ provides a measure of the degree of belongingness of an element

Table 4
Recent applications of AHP method with their details in various studies.

	Author(s)	Year	Uncertainty	Application area
[16]	Chen et al.	2022	Fuzzy set	Supplier selection problem
[78]	Ilbahar et al.	2022	Fuzzy set	Risk assessment of renewable energy investments
[79]	Duleba et al.	2021	Fuzzy set	Public transport development decision-making problem
[18]	Dogan et al.	2020	Fuzzy set	Autonomous vehicles provide selection
[80]	Karacan et al.	2020	Fuzzy set	Chickpea cultivar selection in difficult conditions
[81]	Ar, I.K. et al.	2020	Fuzzy set	Logistics operations evaluation using blockchain
[15]	Seker et al.	2019	Fuzzy set	Solar power plants location selection
[82]	Büyüközkan, G. et al.	2019	Fuzzy set	Digital transformation in the hospitality industry
[83]	Nirmala et al.	2019	Fuzzy set	Supplier selection problem
[84]	Büyüközkan, G. et al.	2019	Fuzzy set	Hazardous waste carriers selection
[85]	Taherkhani et al.	2019	Fuzzy set	Kidney allocation problem
[86]	Yu, Y. et al.	2018	Fuzzy set	Risk factors in water supply projects
[87]	Samanlioglu et al.	2018	Fuzzy set	Information technology (IT) departmental personnel selection process
[19]	Ouyang et al.	2018	Fuzzy set	Municipal wastewater treatment for choosing the paradigms of mangroves
[88]	Atalay et al.	2018	Fuzzy set	Ratio analysis for new product selection
[89]	Hinduja et al.	2018	Fuzzy set	Life insurance product selection
[90]	Wang et al.	2018	Fuzzy set	An assessment of Lhasa's human settlement

y to a fuzzy set ($\tilde{A}_{FS}$). If MV $\mu_{\tilde{A}_{FS}}(y)$ is 0, then y is does not belong to the fuzzy set A_{FS} and if $\mu_{\tilde{A}_{FS}}(y)$ is 1, then y is entirely belong to the fuzzy set $\tilde{A}_{FS}$. The MV $\mu_{\tilde{A}_{FS}}(y)$ lies between $(0,1)$ implies that the element y belongs to the fuzzy set $\tilde{A}_{FS}$ is partially. In case of classical set theory if $\mu_{\tilde{A}_{FS}}(y)$ has a value between 0&1, y is slight pertain to the fuzzy set $\tilde{A}_{FS}$.

There are different shape of MF like triangular [35], trapezoidal [36], pentagonal [37], hexagonal [27], spherical [34], Pythagorean [33], etc.

Definition 2 (*Fuzzy Number*)**.** Fuzzy Number [128] is a special types of Fuzzy Set $\tilde{A}_{FS}$ on the set of Real numbers ($\mathbb{R}$) whose membership function (MF) ($\mu_{\tilde{A}_{FS}}$) define as $\mu_{\tilde{A}_{FS}}: \mathbb{R} \rightarrow [0,1]$ and satisfies following conditions:

I. $\tilde{A}_{FS}$ is normal; i.e., there exist $x \in \mathbb{R}$ such that $\mu_{\tilde{A}_{FS}}(x) = 1$.
II. $\tilde{A}_{FS}$ is convex fuzzy set.
III. The MF $\mu_{\tilde{A}_{FS}}(x)$ is piece-wise continuous.
IV. Support of $\tilde{A}_{FS}$ is bounded.

Representation of Triangular Fuzzy Number (TFN) ($\tilde{A}_{FS}$) presented geometrically in Fig. 2. This figure shows the structural path of its membership function $\mu(x)$ which always belongs to $[0,1]$.

3.2. Intuitionistic fuzzy set

Atanassov, K.T. [129] defines the extension of the fuzzy set to the Intuitionistic Fuzzy Set (IFS) in 1986. The basic definitions of IFS are defined in this section which is applied in further studies. Several studies applied IFS, including decision-making, logic programming, evaluation functions, medical diagnostics, and preference relationships [30]. IFS has two membership functions to capture more uncertainty of the data which makes it more efficient compared with the fuzzy set.

An intuitionistic fuzzy number (IFN) is described by Zhang and Yu [130] as $S = (\mu_S, \nu_S, \pi_S)$ where $\mu_S \in [0,1], \nu_S \in [0,1]$; $0 \leq \mu_S + \nu_S \leq 1$, and the indeterminacy membership function (π_S) determine by $\pi_S = 1 - \mu_S - \nu_S$. The concept of IFS [26,131] is defined as follows:

Table 5
Recent studies of TOPSIS method in different applications.

	Author(s)	Year	Uncertainty	Application area
[94]	Bilgili, F. et al.	2022	Fuzzy set	Evaluate the renewable energy for sustainable development
[95]	Gu, X.B. et al.	2022	Fuzzy set	Risk assessment evaluation for landslide hazards
[96]	Ziquan, X. et al.	2021	Fuzzy set	Check the risk factors for occupational health and safety Cruise Ship Construction
[97]	Nakiboglu, G. et al.	2021	Fuzzy set	Supplier selection problem
[20]	Liu, S. et al.	2021	Fuzzy set	Physical education teaching quality evaluation
[98]	Rouyendegh et al.	2020	Fuzzy set	Green supplier selection assessment
[99]	Faghih-Roohi et al.	2020	Fuzzy set	Pharmaceutical product shipping lanes selection
[100]	Kilic et al.	2020	Fuzzy set	Green supplier selection problem
[101]	Rouyendegh et al.	2020	Fuzzy set	Evaluate the performance of retail industry
[102]	Zhang et al.	2020	Fuzzy set	Complicated and flexible problems with bone transplant selection
[103]	Abdullah et al.	2020		Flood management system estimation
[104]	Memari et al.	2019	Fuzzy set	Investigation of sustainable supplier selection
[105]	Cavallaro et al.	2019	Fuzzy set	Technological forecasting and social change of concentrated solar power technology
[106]	Kansal et al.	2019	Fuzzy set	Patch management software efficiency evaluation
[107]	El Hachami et al.	2019	Fuzzy set	Evaluation of Islamic banking sector contracts
[108]	Rouyendegh et al.	2018	Fuzzy set	Wind power plants site selection
[109]	Pahari et al.	2018	Fuzzy set	Hotel selection process through online review system
[110]	Sen, D.K. et al.	2018	Fuzzy set	Selection problem of sustainable supplier
[111]	Shen, F. et al.	2018	Fuzzy set	Application of credit risk evaluation

Definition 3 (*Intuitionistic Fuzzy Set (IFS)*)**.** Let X be a universal set of discourse. The following is an IFS $\tilde{A}_{IFS}$ on X:

$$\tilde{A}_{IFS} = \left\{ \left\langle x, \mu_{\tilde{A}_{IFS}}(x), \nu_{\tilde{A}_{IFS}}(x) \right\rangle : x \in X \right\} \tag{2}$$

where $\mu_{\tilde{A}_{IFS}} : X \to [0,1]$; $\nu_{\tilde{A}_{IFS}} : X \to [0,1]$ with the condition: $0 \le \mu_{\tilde{A}_{IFS}}(x) + \nu_{\tilde{A}_{IFS}}(x) \le 1$, for all x in $\tilde{A}_{IFS}$.

The set of all intuitionistic fuzzy sets in X shall be denoted by $IFS(X)$. The degrees of MF and non membership function (NMF) of the element x in the fuzzy set $(\tilde{A}_{IFS})$ are denoted by $\mu_{\tilde{A}_{IFS}}(x)$ and $\nu_{\tilde{A}IFS}(x)$, respectively.

Definition 4 (*Degree of Hesitation*)**.** Consider IFS defined in Definition 3. Then the degree of hesitation $(\pi_{\tilde{A}_{IFS}}(x))$ define as

$$\begin{aligned} &\pi_{\tilde{A}_{IFS}}(x) : X \to [0,1] \\ &\pi_{\tilde{A}_{IFS}}(x) = 1 - \mu_{\tilde{A}_{IFS}}(x) - \nu_{\tilde{A}_{IFS}}(x) \end{aligned} \tag{3}$$

for an arbitrary element x in the set $\tilde{A}_{IFS}$. The degree of hesitation is also known as the intuitionistic index or non-determinacy index. For any $x \in X$, it follows naturally that $0 \le \pi_{\tilde{A}_{IFS}}(x) \le 1$.

Presentation of Intuitionistic Fuzzy Number (IFN) $\tilde{A}_{FS}$ and the membership function $\mu(x)$ and non-membership function $\nu(x)$ of the IFN are shown in Fig. 3, which always belongs to $[0,1]$ and $0 \le \mu(x) + \nu(x) \le 1$ for all $x \in X$. Here we see that the shape of the membership function and non-membership function of IFN is triangular.

Table 6
Recent application of IVIF AHP and IVIF TOPSIS techniques in different studies.

	Author(s)	Year	Application area	IVIF-AHP	IVIF-TOPSIS
[112]	Yildiz et al.	2022	Safest route determination for operations of cash in transit	Yes	Yes
[113]	Alimohammadlou et al.	2022	Investigating organizational sustainable development	Yes	No
[114]	Perçin et al.	2022	Circular supplier selection	Yes	No
[115]	Wang et al.	2021	Teaching Effect Evaluation of College English	No	Yes
[116]	Verma et al.	2021	Impact of security attributes determination	Yes	No
[117]	Ayyildiz et al.	2021	Green supply chain resilience evaluation	Yes	No
[118]	Dogan et al.	2020	Selection of corridor for locating autonomous vehicles	Yes	Yes
[119]	Kahraman et al.	2020	Analysis the outsource producers	Yes	Yes
[120]	Abdullah et al.	2020	Protection from flood	No	Yes
[121]	Büyüközkan, G. et al.	2020	Service quality survey and strategic analysis in aviation industry	Yes	No
[122]	Hajek et al.	2019	Effective group decision making	No	Yes
[123]	Rani et al.	2019	Smartphone Selection Problem	No	Yes
[24]	Büyüközkan, G. et al.	2018	Selection of cloud computing innovation	Yes	No
[124]	Tooranloo et al.	2018	Supplier evaluation and selection	Yes	Yes

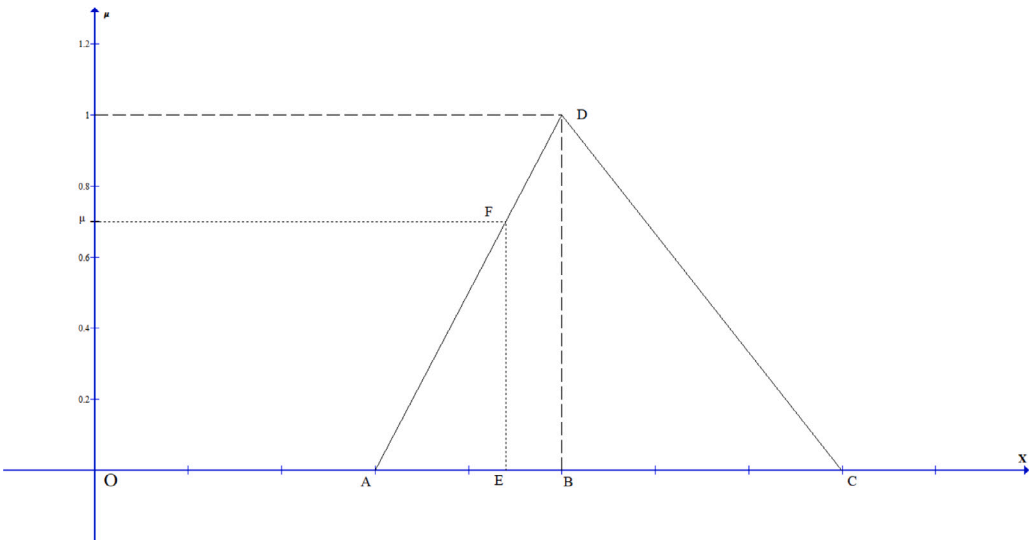


Fig. 2. Representation of membership function ($\mu(x) \in [0, 1]$).

3.3. Interval valued intuitionistic fuzzy set (IVIFS)

Atanasov and Gargov [132] introduced interval-valued intuitionistic fuzzy sets (IVIFS) as a more conventional form of IFS in 1989 after defining the concept of IFSs in 1986. To deal with real world problems, considering the ambiguity in decision-making challenges in that situations. The decision expert expresses their point of view on prioritization values with IFNs [124] on an interval basis, which is sometimes necessary.

Definition 5 (*Interval-Valued Intuitionistic Fuzzy Set (IVIFS) [132,133]*)**.** Consider $D \subseteq [0, 1]$ be the collection of all closed sub-intervals and X be the universal set of discourse. An interval-valued intuitionistic fuzzy set (IVIFS) [134] denoted by $\tilde{B}_{IVIF}$ over X and define

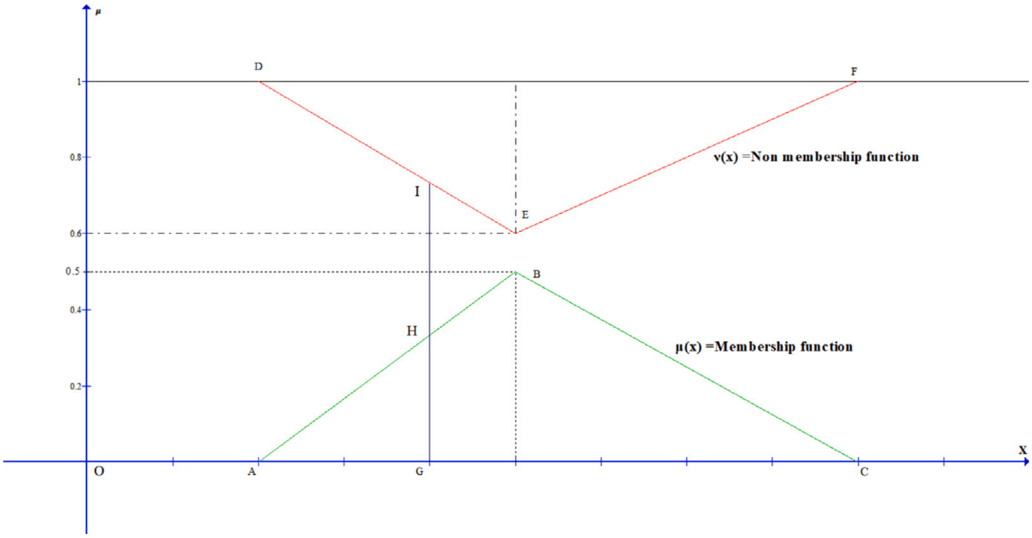


Fig. 3. Geometric presentation of Intuitionistic Fuzzy Number (IFN).

as follows:

$$\tilde{B}_{IVIF} = \left\{ \left\langle y, \mu_{\tilde{B}_{IVIF}}(y), \nu_{\tilde{B}_{IVIF}}(y) \right\rangle : y \in X \right\} \tag{4}$$

where $\mu_{\tilde{B}_{IVIF}}(y) : X \to D \subseteq [0,1], \nu_{\tilde{B}_{IVIF}}(y) : X \to D \subseteq [0,1]$ with the condition
$0 \leq \sup\{\mu_{\tilde{B}_{IVIF}}(y)\} + \sup\{\nu_{\tilde{B}_{IVIF}}(y)\} \leq 1; \forall y \in X.$

The intervals $\mu_{\tilde{B}_{IVIF}}(y)$ and $\nu_{\tilde{B}_{IVIF}}(y)$ represent the membership function (MF) and non membership function (NMF) of the element y in the set $\tilde{B}_{IVIF}$, respectively. As a result, for each element $y \in X$, $\mu_{\tilde{B}_{IVIF}}(y)$ and $\nu_{\tilde{B}_{IVIF}}(y)$ are closed intervals, and their starting and ending points are designated by $\mu^L_{\tilde{B}_{IVIF}}(y)$, $\mu^U_{\tilde{B}_{IVIF}}(y)$, $\nu^L_{\tilde{B}_{IVIF}}(y)$ and $\nu^U_{\tilde{B}_{IVIF}}(y)$, respectively. The interval-valued intuitionistic fuzzy set $\tilde{B}_{IVIF}$ is then denoted by

$$\tilde{B}_{IVIF} = \left\{ \langle y, \left[\mu^L_{\tilde{B}_{IVIF}}(y), \mu^U_{\tilde{B}_{IVIF}}(y)\right], \left[\nu^L_{\tilde{B}_{IVIF}}(y), \nu^U_{\tilde{B}_{IVIF}}(y)\right] \rangle : y \in X \right\} \tag{5}$$

where $\mu^U_{\tilde{B}_{IVIF}}(y) \leq 1$, $\nu^U_{\tilde{B}_{IVIF}}(y) \leq 1$, $\mu^L_{\tilde{B}_{IVIF}}(y) \geq 0$, $\nu^L_{\tilde{B}_{IVIF}}(y) \geq 0$ and $0 \leq \mu^U_{\tilde{B}_{IVIF}}(y) + \nu^U_{\tilde{B}_{IVIF}}(y) \leq 1$.

Given an interval-valued intuitionistic fuzzy set $\tilde{B}_{IVIF}$, we may calculate the unknown degree (hesitancy degree) for each element y in $\tilde{B}_{IVIF}$, defined as:

$$\begin{aligned} \pi_{\tilde{B}_{IVIF}}(y) &= 1 - \mu_{\tilde{B}_{IVIF}}(y) - \nu_{\tilde{B}_{IVIF}}(y) \\ &= \left(\left[1 - \mu^U_{\tilde{B}_{IVIF}}(y) - \nu^U_{\tilde{B}_{IVIF}}(y)\right], \left[1 - \mu^L_{\tilde{B}_{IVIF}}(y) - \nu^L_{\tilde{B}_{IVIF}}(y)\right] \right) \end{aligned} \tag{6}$$

Comparative analysis between three sets FS, IFS and IVIFS are shown in Table 7. The advantages and disadvantages [135–137] of those sets are described as follows:

The graph of IVIF numbers ($\tilde{Q}_{IVIF}$) is shown in Fig. 4. Also, see that the membership function ($\mu_{\tilde{Q}_{IVIF}}$) and non-membership function ($\nu_{\tilde{Q}_{IVIF}}$) are interval valued continuous functions whit triangular shapes.

3.4. Some basic set & arithmetic operation on IVIF sets

This section discusses some basic operations of IVIFS, firstly showing the set operations on IVIFSs and then arithmetic operations on IVIFNs. Consider two IVIFSs $\tilde{A}_{IVIF}$ and $\tilde{B}_{IVIF}$, defined as $\tilde{A}_{IVIF} = \left(\mu_{\tilde{A}_{IVIF}}(x), \nu_{\tilde{A}_{IVIF}}(x)\right) = \left(\left[\mu^L_{\tilde{A}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\right], \left[\nu^L_{\tilde{A}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\right]\right)$ where $\mu_{\tilde{A}_{IVIF}}(x) = \left[\mu^L_{\tilde{A}_{IVIF}}(x), \mu^U_{\tilde{A}_{IVIF}}(x)\right] = \left[\mu^L_{\tilde{A}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\right]$ and $\nu_{\tilde{A}_{IVIF}}(x) = \left[\nu^L_{\tilde{A}_{IVIF}}(x), \nu^U_{\tilde{A}_{IVIF}}(x)\right] = \left[\nu^L_{\tilde{A}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\right]$ and $\tilde{B}_{IVIF} = \left(\mu_{\tilde{B}_{IVIF}}(x), \nu_{\tilde{B}_{IVIF}}(x)\right) = \left(\left[\mu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{B}_{IVIF}}\right], \left[\nu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{B}_{IVIF}}\right]\right)$ where $\mu_{\tilde{B}_{IVIF}}(x) = \left[\mu^L_{\tilde{B}_{IVIF}}(x), \mu^U_{\tilde{B}_{IVIF}}(x)\right] = \left[\mu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{B}_{IVIF}}\right]$ and $\nu_{\tilde{B}_{IVIF}}(x) = \left[\nu^L_{\tilde{B}_{IVIF}}(x), \nu^U_{\tilde{B}_{IVIF}}(x)\right] = \left[\nu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{B}_{IVIF}}\right]$, respectively. We utilize the following operations to aggregate IVIF numbers [32,138].

Definition 6 (*Set Operation [32,139]*). Let $\tilde{A}_{IVIF} = \left(\left[\mu^L_{\tilde{A}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\right], \left[\nu^L_{\tilde{A}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\right]\right)$ and $\tilde{B}_{IVIF} = \left(\left[\mu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{B}_{IVIF}}\right], \left[\nu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{B}_{IVIF}}\right]\right)$ be two interval-valued intuitionistic fuzzy sets (IVIFSs). Then

Table 7
Advantages & disadvantages of FS, IFS & IVIFs on various parameters.

Different characteristics	Fuzzy set	Intuitionistic fuzzy set	Interval-valued intuitionistic fuzzy set
Membership/ Non-Membership degree	Only Membership degree	Both Membership degree & Non-Membership degree	Both Membership degree & Non-Membership degree with interval format
Type of Membership value	Fixed real-valued	Fixed real-valued	Fixed interval real valued
Preciseness	Value of parameter not precise	More precise	Most Precise
Hesitation Degree	Cannot find out	Can find out	Can find out
Boundaries	Fixed	Fixed	Interval-valued fixed boundaries
Accuracy & Efficiency	Less	Average	High
Belongingness & Non-belongingness	Measure the belongingness only	Measure the belongingness & Non belongingness both	Measure the belongingness & Non belongingness in interval-valued format
Decision Making/ Judgment	Suitable	More suitable	Most suitable for robust & effective decision
Computational Method	Very easy	Easy	Little complex

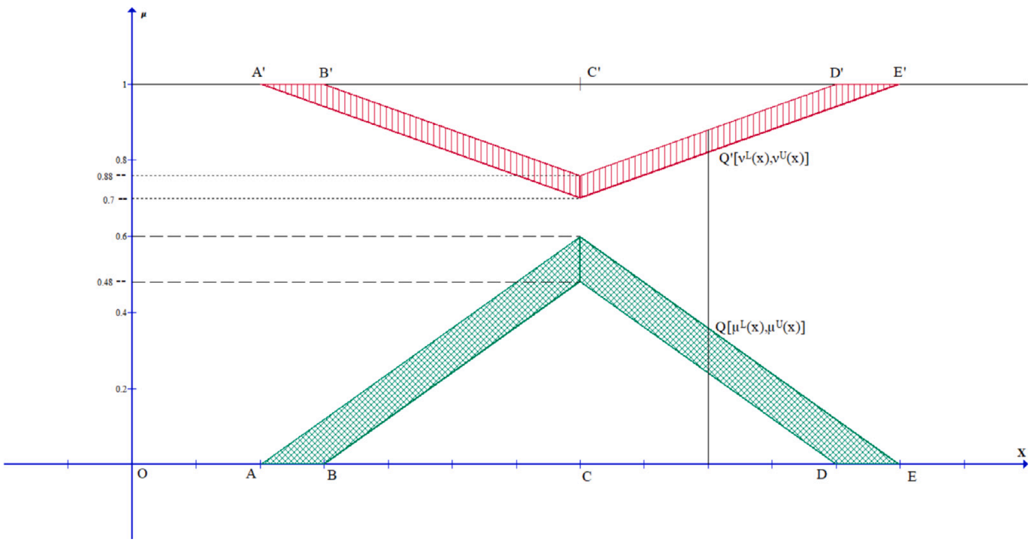


Fig. 4. Geometric presentation of IVIF number ($\tilde{Q}_{IVIF}$).

I. Union of two IVIFSs:

$$\tilde{A}_{IVIF} \cup \tilde{B}_{IVIF} = \left(\left[max\left\{\mu^L_{\tilde{A}_{IVIF}}, \mu^L_{\tilde{B}_{IVIF}}\right\}, max\left\{\mu^R_{\tilde{A}_{IVIF}}, \mu^R_{\tilde{B}_{IVIF}}\right\}\right], \left[min\left\{\nu^L_{\tilde{A}_{IVIF}}, \nu^L_{\tilde{B}_{IVIF}}\right\}, min\left\{\nu^R_{\tilde{A}_{IVIF}}, \nu^R_{\tilde{B}_{IVIF}}\right\}\right]\right) \tag{7}$$

II. Intersection of two IVIFSs:

$$\tilde{A}_{IVIF} \cap \tilde{B}_{IVIF} = \left(\left[min\left\{\mu^L_{\tilde{A}_{IVIF}}, \mu^L_{\tilde{B}_{IVIF}}\right\}, min\left\{\mu^R_{\tilde{A}_{IVIF}}, \mu^R_{\tilde{B}_{IVIF}}\right\}\right], \left[max\left\{\nu^L_{\tilde{A}_{IVIF}}, \nu^L_{\tilde{B}_{IVIF}}\right\}, max\left\{\nu^R_{\tilde{A}_{IVIF}}, \nu^R_{\tilde{B}_{IVIF}}\right\}\right]\right) \tag{8}$$

III. Complement of IVIFS:

$$\tilde{A}^C_{IVIF} = \left(\left[\nu^L_{\tilde{A}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\right], \left[\mu^L_{\tilde{A}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\right]\right) \tag{9}$$

IV. Subset of IVIFS:

$$\tilde{A}_{IVIF} \subseteq \tilde{B}_{IVIF} \iff \begin{cases} \mu^L_{\tilde{A}_{IVIF}} \le \mu^L_{\tilde{B}_{IVIF}}, & \mu^U_{\tilde{A}_{IVIF}} \le \mu^U_{\tilde{B}_{IVIF}} \\ & \& \\ \nu^L_{\tilde{A}_{IVIF}} \ge \nu^L_{\tilde{B}_{IVIF}}, & \nu^U_{\tilde{A}_{IVIF}} \ge \nu^U_{\tilde{B}_{IVIF}} \end{cases} \tag{10}$$

IV. Equality of two IVIFSs:

$$\tilde{A}_{IVIF} = \tilde{B}_{IVIF} \iff \tilde{A}_{IVIF} \subseteq \tilde{B}_{IVIF} \ \& \ \tilde{B}_{IVIF} \subseteq \tilde{A}_{IVIF} \tag{11}$$

Definition 7 (*Arithmetic Operation [139,140]*). Let $\tilde{A}_{IVIF} = \left(\left[\mu^L_{\tilde{A}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\right], \left[\nu^L_{\tilde{A}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\right]\right)$ and $\tilde{B}_{IVIF} = \left(\left[\mu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{B}_{IVIF}}\right], \left[\nu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{B}_{IVIF}}\right]\right)$ be two interval-valued intuitionistic fuzzy numbers (IVIFNs) and $\lambda \in \mathbb{R}$ be an arbitrary positive real number. Then

I. Addition of two IVIFNs:

$$\tilde{A}_{IVIF} \oplus \tilde{B}_{IVIF} = \left(\left[\mu^L_{\tilde{A}_{IVIF}} + \mu^L_{\tilde{B}_{IVIF}} - \mu^L_{\tilde{A}_{IVIF}}\mu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}} + \mu^U_{\tilde{B}_{IVIF}} - \mu^U_{\tilde{A}_{IVIF}}\mu^U_{\tilde{B}_{IVIF}}\right], \left[\nu^L_{\tilde{A}_{IVIF}}\nu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\nu^U_{\tilde{B}_{IVIF}}\right]\right) \tag{12}$$

II. Multiplication of two IVIFNs:

$$\tilde{A}_{IVIF} \otimes \tilde{B}_{IVIF} = \left(\left[\mu^L_{\tilde{A}_{IVIF}}\mu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\mu^U_{\tilde{B}_{IVIF}}\right], \left[\nu^L_{\tilde{A}_{IVIF}} + \nu^L_{\tilde{B}_{IVIF}} - \nu^L_{\tilde{A}_{IVIF}}\nu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}} + \nu^U_{\tilde{B}_{IVIF}} - \nu^U_{\tilde{A}_{IVIF}}\nu^U_{\tilde{B}_{IVIF}}\right]\right) \tag{13}$$

III. Scalar Multiplication of IVIFNs:

$$\lambda\tilde{A}_{IVIF} = \left(\left[1 - \left(1 - \mu^L_{\tilde{A}_{IVIF}}\right)^\lambda, 1 - \left(1 - \mu^U_{\tilde{A}_{IVIF}}\right)^\lambda\right], \left[\left(\nu^L_{\tilde{A}_{IVIF}}\right)^\lambda, \left(\nu^U_{\tilde{A}_{IVIF}}\right)^\lambda\right]\right) \tag{14}$$

IV. Power of IVIFN:

$$\tilde{A}^\lambda_{IVIF} = \left(\left[\left(\mu^L_{\tilde{A}_{IVIF}}\right)^\lambda, \left(\mu^U_{\tilde{A}_{IVIF}}\right)^\lambda\right]\right), \left(\left[1 - \left(1 - \nu^L_{\tilde{A}_{IVIF}}\right)^\lambda, 1 - \left(1 - \nu^U_{\tilde{A}_{IVIF}}\right)^\lambda\right]\right) \tag{15}$$

V. Inverse of IVIFN:

$$\tilde{A}^{-1}_{IVIF} = \left(\left[\nu^L_{\tilde{A}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\right], \left[\mu^L_{\tilde{A}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\right]\right) \tag{16}$$

VI. Deviation of two IVIF :

$$\tilde{A}_{IVIF} \oslash \tilde{B}_{IVIF} = \left(\left[\mu^L_{\tilde{A}_{IVIF}}\nu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\nu^U_{\tilde{B}_{IVIF}}\right], \left[\nu^L_{\tilde{A}_{IVIF}} + \mu^L_{\tilde{B}_{IVIF}} - \nu^L_{\tilde{A}_{IVIF}}\mu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}} + \mu^U_{\tilde{B}_{IVIF}} - \nu^U_{\tilde{A}_{IVIF}}\mu^U_{\tilde{B}_{IVIF}}\right]\right) \tag{17}$$

Theorem 1 (*[139]*). *Let* $\tilde{A}_{IVIF} = \left(\left[\mu^L_{\tilde{A}_{IVIF}}, \mu^U_{\tilde{A}_{IVIF}}\right], \left[\nu^L_{\tilde{A}_{IVIF}}, \nu^U_{\tilde{A}_{IVIF}}\right]\right)$, $\tilde{B}_{IVIF} = \left(\left[\mu^L_{\tilde{B}_{IVIF}}, \mu^U_{\tilde{B}_{IVIF}}\right], \left[\nu^L_{\tilde{B}_{IVIF}}, \nu^U_{\tilde{B}_{IVIF}}\right]\right)$ *and* $\tilde{C}_{IVIF} = \left(\left[\mu^L_{\tilde{C}_{IVIF}}, \mu^U_{\tilde{C}_{IVIF}}\right], \left[\nu^L_{\tilde{C}_{IVIF}}, \nu^U_{\tilde{C}_{IVIF}}\right]\right)$ *are three arbitrary interval-valued intuitionistic fuzzy numbers.*

A. Properties of Addition operation on IVIFNs:

I. Closure property: *If* $\tilde{A}_{IVIF}$ *and* $\tilde{B}_{IVIF}$ *are two IVIF numbers, then* $\tilde{A}_{IVIF} \oplus \tilde{B}_{IVIF}$ *be also an IVIF number.*

II. Commutative law:

$$\tilde{A}_{IVIF} \oplus \tilde{B}_{IVIF} = \tilde{B}_{IVIF} \oplus \tilde{A}_{IVIF} \tag{18}$$

III. Associative law:

$$(\tilde{A}_{IVIF} \oplus \tilde{B}_{IVIF}) \oplus \tilde{C}_{IVIF} = \tilde{B}_{IVIF} \oplus (\tilde{A}_{IVIF} \oplus \tilde{C}_{IVIF}) \tag{19}$$

IV. De-Morgan law:

$$\left(\tilde{A}_{IVIF} \oplus \tilde{B}_{IVIF}\right)^{C} = \tilde{A}^{C}_{IVIF} \otimes \tilde{B}^{C}_{IVIF} \tag{20}$$

B. Properties of Multiplication operation:

I. Closure property: *If $\tilde{A}_{IVIF}$ and $\tilde{B}_{IVIF}$ are two IVIF numbers, then $\tilde{A}_{IVIF} \otimes \tilde{B}_{IVIF}$ be also an IVIF number.*

II. Commutative law:

$$\tilde{A}_{IVIF} \otimes \tilde{B}_{IVIF} = \tilde{B}_{IVIF} \otimes \tilde{A}_{IVIF} \tag{21}$$

III. Associative law:

$$(\tilde{A}_{IVIF} \otimes \tilde{B}_{IVIF}) \otimes \tilde{C}_{IVIF} = \tilde{B}_{IVIF} \otimes (\tilde{A}_{IVIF} \otimes \tilde{C}_{IVIF}) \tag{22}$$

IV. De-Morgan law:

$$\left(\tilde{A}_{IVIF} \otimes \tilde{B}_{IVIF}\right)^{C} = \tilde{A}^{C}_{IVIF} \oplus \tilde{B}^{C}_{IVIF} \tag{23}$$

Definition 8 (*Defuzzification of IVIFN [133]*)**.** Let $\tilde{A}_{IVIF} = \left(\left[\mu^{L}_{\tilde{A}_{IVIF}}, \mu^{U}_{\tilde{A}_{IVIF}}\right], \left[\nu^{L}_{\tilde{A}_{IVIF}}, \nu^{U}_{\tilde{A}_{IVIF}}\right]\right)$ be an IVIFN. Defuzzification of $\tilde{A}_{IVIF}$ is proposed using the scoring formula below

$$I\left(\tilde{A}_{IVIF}\right) = \frac{\mu^{L}_{\tilde{A}_{IVIF}} + \mu^{U}_{\tilde{A}_{IVIF}} + \left(1-\nu^{L}_{\tilde{A}_{IVIF}}\right) + \left(1-\nu^{U}_{\tilde{A}_{IVIF}}\right) + \mu^{L}_{\tilde{A}_{IVIF}} \times \mu^{U}_{\tilde{A}_{IVIF}} - \sqrt{\left(1-\nu^{L}_{\tilde{A}_{IVIF}}\right)\left(1-\nu^{U}_{\tilde{A}_{IVIF}}\right)}}{4} \tag{24}$$

Non-membership degrees are converted to membership degrees by the terms $\left(1-\nu^{L}_{\tilde{A}_{IVIF}}\right)$ and $\left(1-\nu^{U}_{\tilde{A}_{IVIF}}\right)$ respectively, whereas the defuzzified value is decreased by the expression $\sqrt{\left(1-\nu^{L}_{\tilde{A}_{IVIF}}\right)\left(1-\nu^{U}_{\tilde{A}_{IVIF}}\right)}$.

Definition 9 (*Score Function [141]*)**.** Let an interval-valued intuitionistic fuzzy number (IVIFN) like $\tilde{A}_{IVIF} = \left(\left[\mu^{-}_{\tilde{A}_{IVIF}}, \mu^{+}_{\tilde{A}_{IVIF}}\right], \left[\nu^{-}_{\tilde{A}_{IVIF}}, \nu^{+}_{\tilde{A}_{IVIF}}\right]\right)$ exist. The following is a representation of the score function for ranking interval-valued intuitionistic fuzzy values:

$$S\left(\tilde{A}_{IVIF}\right) = \frac{\mu^{-}_{\tilde{A}_{IVIF}} + \mu^{+}_{\tilde{A}_{IVIF}} - \nu^{-}_{\tilde{A}_{IVIF}} - \nu^{+}_{\tilde{A}_{IVIF}}}{2}. \tag{25}$$

Definition 10 (*Accuracy Function [141]*)**.** Let $\tilde{A}_{IVIF} = \left(\left[\mu^{-}_{\tilde{A}_{IVIF}}, \mu^{+}_{\tilde{A}_{IVIF}}\right], \left[\nu^{-}_{\tilde{A}_{IVIF}}, \nu^{+}_{\tilde{A}_{IVIF}}\right]\right)$ be an IVIF number. To measure the IVIF number $\tilde{A}_{IVIF}$, Xu devised the following accuracy function:

$$H\left(\tilde{A}_{IVIF}\right) = \frac{\mu^{-}_{\tilde{A}_{IVIF}} + \mu^{+}_{\tilde{A}_{IVIF}} + \nu^{-}_{\tilde{A}_{IVIF}} + \nu^{+}_{\tilde{A}_{IVIF}}}{2}. \tag{26}$$

Note 1. *The score function is used to measure the interval-valued intuitionistic fuzzy number. Let $\tilde{A}_{IVIF}$ and $\tilde{B}_{IVIF}$ be two IVIF number, if $S\left(\tilde{A}_{IVIF}\right) < S\left(\tilde{B}_{IVIF}\right)$ then $\tilde{A}_{IVIF} < \tilde{B}_{IVIF}$. The accuracy function will be used to determine the ranking of IVIF numbers if the score values of any two IVIF numbers are equal.*

Definition 11 (*Hamming Distance [142]*)**.** Let $\tilde{A}_{IVIF} = \left(\left[\mu^{-}_{\tilde{A}_{IVIF}}, \mu^{+}_{\tilde{A}_{IVIF}}\right], \left[\nu^{-}_{\tilde{A}_{IVIF}}, \nu^{+}_{\tilde{A}_{IVIF}}\right]\right)$ and $\tilde{B}_{IVIF} = \left(\left[\mu^{-}_{\tilde{B}_{IVIF}}, \mu^{+}_{\tilde{B}_{IVIF}}\right], \left[\nu^{-}_{\tilde{B}_{IVIF}}, \nu^{+}_{\tilde{B}_{IVIF}}\right]\right)$ be two IVIF numbers. The Hamming distance is used to calculate the distance between these two IVIF numbers, as shown in Eq. (27).

$$HD = \frac{1}{4}\left(\|\mu^{-}_{\tilde{A}_{IVIF}} - \mu^{-}_{\tilde{B}_{IVIF}}\| + \|\mu^{+}_{\tilde{A}_{IVIF}} - \mu^{+}_{\tilde{B}_{IVIF}}\| + \|\nu^{-}_{\tilde{A}_{IVIF}} - \nu^{-}_{\tilde{B}_{IVIF}}\| + \|\nu^{+}_{\tilde{A}_{IVIF}} - \nu^{+}_{\tilde{B}_{IVIF}}\|\right) \tag{27}$$

Definition 12. Let, $\tilde{\beta}_j = ([\mu_j^-, \mu_j^+], [\nu_j^-, \nu_j^+])$ $(j = 1, 2, 3, \ldots, n)$ be the set of IVIFNs and then the operator $Q^n \rightarrow Q$ is called the interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operator, if

$$IIFWA_w(\tilde{\beta}_1, \tilde{\beta}_2, \ldots., \tilde{\beta}_n) = w_1\tilde{\beta}_1 \oplus w_2\tilde{\beta}_2 \oplus w_n\tilde{\beta}_n \tag{28}$$

where Q is the collection of all IVIFNs, $w = (w_1, w_2, \ldots .w_n)$ is the weight vector of the IVIFNs $\tilde{\beta}_j$ $(j = 1, 2, \ldots, n)$ and $w_j \geq 0$, $\sum_{j=1}^{n} w_j = 1$. Further, this IVIFWA operator can be modified to look like this [21].

$$IIFWA_w(\tilde{\beta}_1, \tilde{\beta}_2, \ldots., \tilde{\beta}_n) = \left(\left[1 - \left(\prod_{j=1}^{n}(1-\mu_j^-)\right)^{w_j}, 1 - \left(\prod_{j=1}^{n}(1-\mu_j^+)\right)^{w_j}\right], \left[\left(\prod_{j=1}^{n}\nu_j^-\right)^{w_j}, \left(\prod_{j=1}^{n}\nu_j^+\right)^{w_j}\right]\right) \tag{29}$$

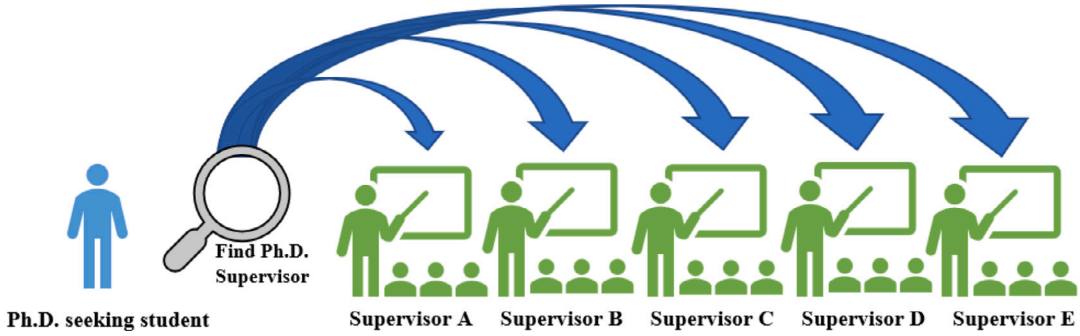


Fig. 5. Student searching for PhD supervisor.

When the weights of the decision maker are equal, i.e., $w = (1/n, 1/n, 1/n, \ldots, 1/n)$ then the IVIFWA operator decreases to an interval-valued intuitionistic fuzzy averaging (IVIFA) operator, where

$$\begin{aligned} IIFA(\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_n) &= \frac{1}{n}(\tilde{\beta}_1 \oplus \tilde{\beta}_2 \oplus \ldots \oplus \tilde{\beta}_n) \\ &= \left(\left[1 - \left(\prod_{j=1}^{n} (1 - \mu_j^-) \right)^{1/n}, 1 - \left(\prod_{j=1}^{n} (1 - \mu_j^+) \right)^{1/n} \right], \left[\left(\prod_{j=1}^{n} \nu_j^- \right)^{1/n}, \left(\prod_{j=1}^{n} \nu_j^+ \right)^{1/n} \right] \right) \end{aligned} \tag{30}$$

4. Interval-valued intuitionistic MCDM methodology

MCDM with fuzzy uncertainty with applications are not new in the research field (see for example [143–146]). We present a technique for MCDM problems using an IVIF AHP and the IVIF TOPSIS method in this paper. There are two Phases to the approach. The first phase is of eight steps that culminate in the calculation of the criteria weights. In a similar way, the second phase consists of eight steps and ends with a ranking of the options according to expert opinions and the criteria weights obtained in the first phase.

4.1. Interval-valued intuitionistic fuzzy AHP

There are numerous frameworks for evaluating and selecting PhD supervisor, and school teachers, each with its own set of criteria, resulting in various scenarios [2,3,5–7,59]. There are both quantitative and qualitative criteria for choosing a PhD supervisor, however, there may be a requirement to combine both quantitative and qualitative criteria within a single framework [147]. IVIFS is a great method for handling ambiguity and uncertainty because each element is given a membership degree value, a non-membership degree value, and an uncertainty degree value. An interval-valued intuitionistic fuzzy Analytic hierarchy process (AHP) model for MCDM problems could be developed. Saaty's proposed AHP method and IVIF set theory are both incorporated into the IVIF approach. Here, Figs. 5 and 6 depict the hierarchical structure of the supervisor consideration and choice model used in this study.

The flow chart of this study is portrayed in Fig. 7. The weight of evaluation criteria $(j = C_1, C_2, \ldots, C_n)$ will be determined first by IVIF AHP [119] in the suggested model. Assume there are m supervisors $A_i(1, \ldots, m)$ who are assessed by l members $M_k(1, \ldots, l)$ using the aforesaid criteria and the IVIF linguistic factors indicated in Table 9. The steps of our proposed method are outlined below as follows:

Step 1: Construct the pairwise comparison matrix:
This step will involve creating a pairwise comparison matrix $\left(\tilde{R} = \left(\tilde{r}_{ij}\right)_{n\times n}\right)$ for Ph.D Supervisor selection criteria, which will be contrasted based on expert judgments. Pairwise comparison matrix and aggregated pairwise comparison matrix will be made using the linguistic variable and IVIF numbers from Table 9.

$$\tilde{R} = \begin{bmatrix} ([\mu_{11}^-, \mu_{11}^+], [\nu_{11}^-, \nu_{11}^+]) & \cdots & ([\mu_{1n}^-, \mu_{1n}^+], [\nu_{1n}^-, \nu_{1n}^+]) \\ \vdots & \ddots & \vdots \\ ([\mu_{n1}^-, \mu_{n1}^+], [\nu_{n1}^-, \nu_{n1}^+]) & \cdots & ([\mu_{nn}^-, \mu_{nn}^+], [\nu_{nn}^-, \nu_{nn}^+]) \end{bmatrix} \tag{31}$$

This is the pairwise comparison matrix between criteria to criteria.

Step 2: Calculating the Score judgment pairwise comparison matrix with score function.

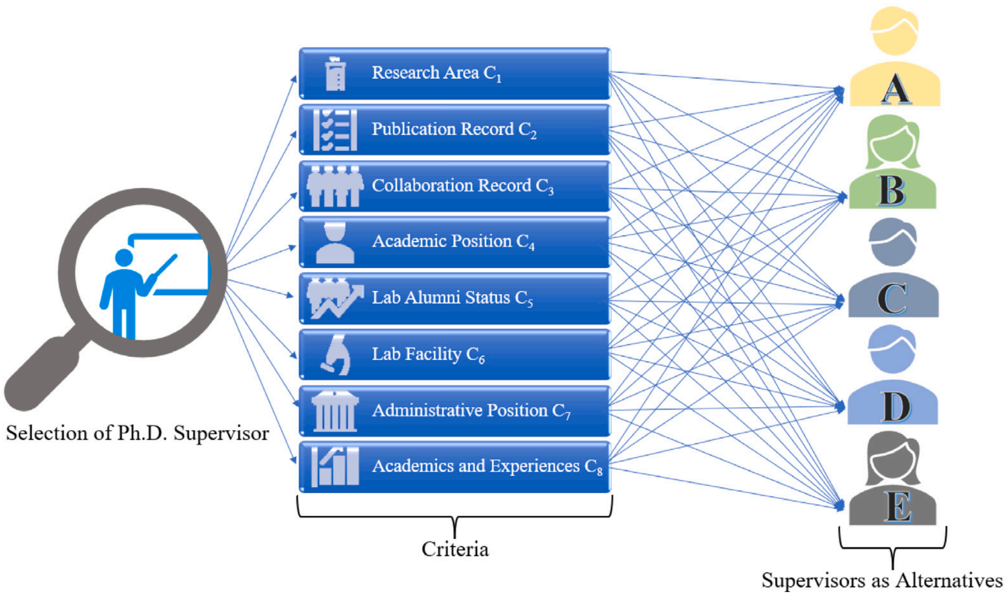


Fig. 6. The Hierarchical structure of MCDM strategy.

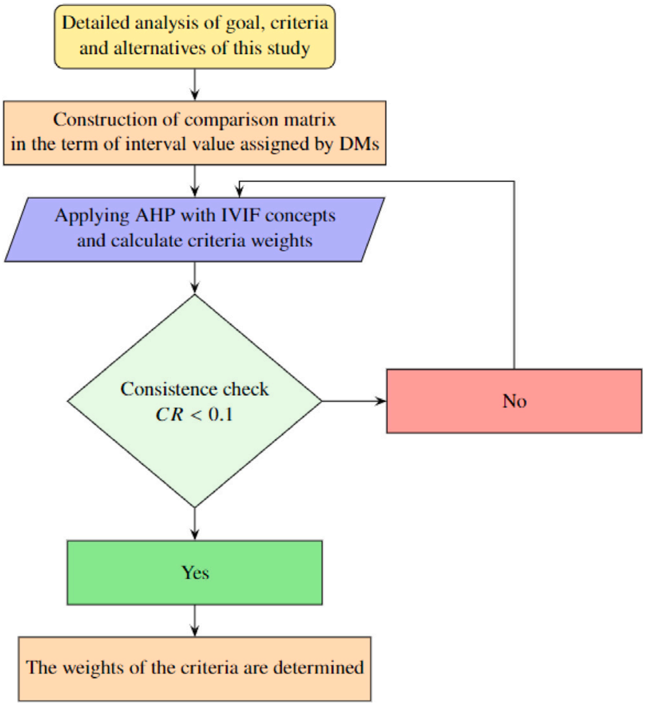


Fig. 7. Flow chart for the IVIF AHP method and calculate weights of the criteria.

Here Eq. (32) determines the score function $(\tilde{s}_{ij})$ of each member of the group pairwise comparison matrix $\left(\tilde{S} = (\tilde{s}_{ij})_{n\times n}\right)$.

$$\tilde{S} = \begin{bmatrix} ([\mu_{11}^{-} - \nu_{11}^{+}], [\mu_{11}^{+} - \nu_{11}^{-}]) & \dots & ([\mu_{1n}^{-} - \nu_{1n}^{+}], [\mu_{1n}^{+} - \nu_{1n}^{-}]) \\ \vdots & \ddots & \vdots \\ ([\mu_{n1}^{-} - \nu_{n1}^{+}], [\mu_{n1}^{+} - \nu_{n1}^{-}]) & \dots & ([\mu_{nn}^{-} - \nu_{nn}^{+}], [\mu_{nn}^{+} - \nu_{nn}^{-}]) \end{bmatrix} \tag{32}$$

where $\tilde{s}_{ij} = ([\mu_{ij}^{-} - \nu_{ij}^{+}], [\mu_{ij}^{+} - \nu_{ij}^{-}])$.

Step 3: Formulate the interval exponential matrices ($\tilde{A}$) as given in Eq. (33), as follows

$$\tilde{A} = \begin{bmatrix} [e^{(\mu_{11}^{-}-\nu_{11}^{+})}, e^{(\mu_{11}^{+}-\nu_{11}^{-})}] & \dots & [e^{(\mu_{1n}^{-}-\nu_{1n}^{+})}, e^{(\mu_{1n}^{+}-\nu_{1n}^{-})}] \\ \vdots & \ddots & \vdots \\ [e^{(\mu_{n1}^{-}-\nu_{n1}^{+})}, e^{(\mu_{n1}^{+}-\nu_{n1}^{-})}] & \dots & [e^{(\mu_{nn}^{-}-\nu_{nn}^{+})}, e^{(\mu_{nn}^{+}-\nu_{nn}^{-})}] \end{bmatrix} = \begin{bmatrix} [\tilde{b}_{11}^{-}, \tilde{b}_{11}^{+}] & \dots & [\tilde{b}_{1n}^{-}, \tilde{b}_{1n}^{+}] \\ \vdots & \ddots & \vdots \\ [\tilde{b}_{n1}^{-}, \tilde{b}_{n1}^{+}] & \dots & [\tilde{b}_{nn}^{-}, \tilde{b}_{nn}^{+}] \end{bmatrix} \tag{33}$$

Step 4: Calculating the priority vector of each criterion using Eq. (34), as follows

$$\tilde{W}_i = \left[\frac{\sum_{j=1}^{n} \tilde{b}_{ij}^{-}}{\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{b}_{ij}^{+}}, \frac{\sum_{j=1}^{n} \tilde{b}_{ij}^{+}}{\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{b}_{ij}^{-}} \right] = \left[\tilde{W}_i^{-}, \tilde{W}_i^{+}\right] \tag{34}$$

where $i = 1, 2, 3, \dots, n$.

Step 5: Construct the possibility degree matrices $\left(P = (p_{ij})_{m\times n}\right)$ by using Eq. (35), as

$$\begin{cases} P\left(\tilde{W}_i > \tilde{W}_j\right) = p_{ij} = \dfrac{\max\left(0, W_i^{+} - W_j^{-}\right) - \max\left(0, W_i^{-} - W_j^{+}\right)}{\left(W_i^{+} - W_i^{-}\right) + \left(W_j^{+} - W_j^{-}\right)} \\ P\left(\tilde{W}_j > \tilde{W}_i\right) = p_{ij} = \dfrac{\max\left(0, W_j^{+} - W_i^{-}\right) - \max\left(0, W_j^{-} - W_i^{+}\right)}{\left(W_i^{+} - W_i^{-}\right) + \left(W_j^{+} - W_j^{-}\right)} \end{cases} \tag{35}$$

Step 6: Possibility degrees are prioritized using Eq. (36), as follows

$$W_i = \frac{\sum_{j=1}^{n} p_{ij} - 1}{n} + 0.5 \tag{36}$$

Step 7: The weights are normalized as given in Eq. (37) and obtain the final weight as

$$W_i^T = \frac{W_i}{\sum_{i=1}^{n} W_i} \tag{37}$$

To get the weights of every criterion and its sub criterion, repeat the steps (1–7). Eq. (37) gives the weight of the criteria and sub-criteria evaluated by IVIF AHP method. Then the second phase will start with step 8 by collecting data from experts' judgment and ends with a prioritization of the alternative.

4.2. Interval-valued intuitionistic fuzzy TOPSIS

In this phase, prioritizing the Ph.D supervisor by Interval-valued intuitionistic fuzzy Technique for Order of Preference by Similarity to Ideal Solution (IVIF TOPSIS) [28,119]. The criteria weights which are obtained by IVIF AHP method, will be used in this phase. The structural flowchart of the TOPSIS methodology is depicted in Fig. 8. The steps of this phase are demonstrated as follows:

Step 8: Construction of the decision matrix:
The interval-valued intuitionistic preferences matrix $R^k = (\tilde{r}_{ij}^{k})_{m\times n}$, should be assigned, where k is the expert opinion by using Table 9.

$$R^k = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \dots & A_m \end{array} \\ \begin{bmatrix} ([\mu_{11k}^{-}, \mu_{11k}^{+}], [\nu_{11k}^{-}, \nu_{11k}^{+}]) & ([\mu_{12k}^{-}, \mu_{12k}^{+}], [\nu_{12k}^{-}, \nu_{12k}^{+}] & \dots & ([\mu_{1mk}^{-}, \mu_{1mk}^{+}], [\nu_{1mk}^{-}, \nu_{1mk}^{+}]) \\ ([\mu_{21k}^{-}, \mu_{21k}^{+}], [\nu_{21k}^{-}, \nu_{21k}^{+}]) & ([\mu_{22k}^{-}, \mu_{22k}^{+}], [\nu_{22k}^{-}, \nu_{22k}^{+}]) & \dots & ([\mu_{2mk}^{-}, \mu_{2mk}^{+}], [\nu_{2mk}^{-}, \nu_{2mk}^{+}]) \\ \vdots & \vdots & \ddots & \vdots \\ ([\mu_{n1k}^{-}, \mu_{n1k}^{+}], [\nu_{n1k}^{-}, \nu_{n1k}^{+}]) & ([\mu_{n2k}^{-}, \mu_{n2k}^{+}], [\nu_{n2k}^{-}, \nu_{n2k}^{+}]) & \dots & ([\mu_{nmk}^{-}, \mu_{nmk}^{+}], [\nu_{nmk}^{-}, \nu_{nmk}^{+}]) \end{bmatrix} \end{array} \tag{38}$$

This is the decision matrix between criteria to alternatives.

Step 9: Aggregating IVIF fuzzy decision matrix:
As previously stated, there are k decision matrices in which m supervisors are rated by k experts based on n criteria. To obtain interval-valued intuitionistic fuzzy decision matrix, aggregate $R^k = (\tilde{r}_{ij}^{k})_{m\times n}$ matrix by using Eq. (30) and aggregated matrix (R) shown as:

$$R = (\tilde{r}_{ij}^{A})_{m\times n} \tag{39}$$

Step 10: The weighted collective interval-valued intuitionistic fuzzy decision matrix $R^* = (\tilde{r}_{ij}^{*})_{m\times n}$ by using Eq. (40).

$$\tilde{r}_{ij}^{*} = W_i \otimes \tilde{r}_{ij}^{A} = \left\langle \left[\mu_{ij}^{-}, \mu_{ij}^{+}\right], \left[\nu_{ij}^{-}, \nu_{ij}^{+}\right] \right\rangle \tag{40}$$

Here the criteria weights are the crisp values, derived from the IVIF AHP technique.

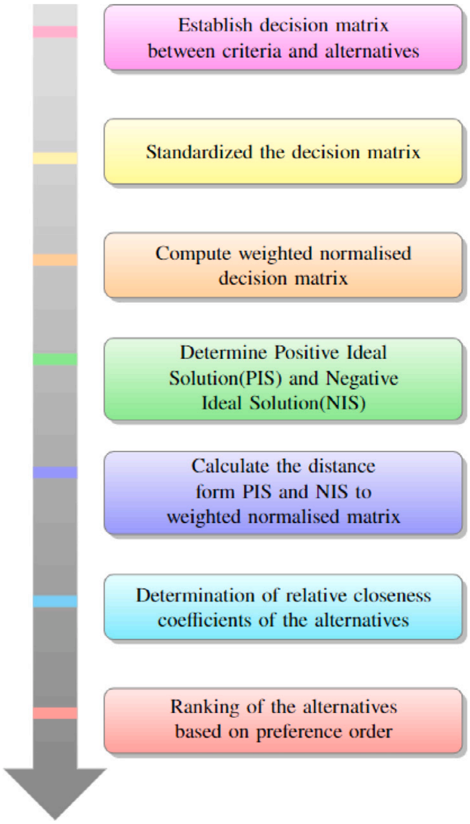


Fig. 8. The schematic diagram of the TOPSIS method.

Step 11: Determining the IVIF Positive Ideal Solution and IVIF Negative Ideal Solution. When the criterion is positive (benefit), Eq. (41) will be used to determine the positive ideal and Eq. (42) will be used to determine the negative ideal, and the criterion is negative (cost), Eq. (42) will be used to determine the positive ideal and Eq. (41) will be used to determine the negative ideal.

The set of benefit criteria and cost criteria are denoted as $B\&C$ respectively. Then, we can determine the IVIF PIS as $A^+ = (r_1^+, r_2^+, \dots, r_n^+)$, where

$$r_j^+ = \begin{cases} \left\langle \left[\max_i \mu_{ij}^-, \max_i \mu_{ij}^+\right], \left[\min_i \nu_{ij}^-, \min_i \nu_{ij}^+\right]\right\rangle, & j \in B, \\ \left\langle \left[\min_i \mu_{ij}^-, \min_i \mu_{ij}^+\right], \left[\max_i \nu_{ij}^-, \max_i \nu_{ij}^+\right]\right\rangle, & j \in C \end{cases} \tag{41}$$

We can also ascertain the IVIF NIS as $A^- = (r_1^-, r_2^-, \dots, r_n^-)$, where

$$r_j^- = \begin{cases} \left\langle \left[\min_i \mu_{ij}^-, \min_i \mu_{ij}^+\right], \left[\max_i \nu_{ij}^-, \max_i \nu_{ij}^+\right]\right\rangle, & j \in B \\ \left\langle \left[\max_i \mu_{ij}^-, \max_i \mu_{ij}^+\right], \left[\min_i \nu_{ij}^-, \min_i \nu_{ij}^+\right]\right\rangle, & j \in C, \end{cases} \tag{42}$$

Without loss of generality we can assume that for $j = 1, 2, \dots, n$, we have $r_j^+ = \left\langle \left[\mu_{j+}^-, \mu_{j+}^+\right], \left[\nu_{j+}^-, \nu_{j+}^+\right]\right\rangle$ and $r_j^- = \left\langle \left[\mu_{j-}^-, \mu_{j-}^+\right], \left[\nu_{j-}^-, \nu_{j-}^+\right]\right\rangle$.

Step 12: Calculate the separation measure:

There are several ways to determine the distance between two IVIF numbers. But here we have used the normalized Hamming distance to define the separation measure degree between alternatives A_i and the IVIF positive-ideal solution described in Eq. (43).

$$d_i^+ = \frac{1}{4m} \sum_{j=1}^{n} \left\{ \left\|\mu_{ij}^- - \mu_{j+}^-\right\| + \left\|\mu_{ij}^+ - \mu_{j+}^+\right\| + \left\|\nu_{ij}^- - \nu_{j+}^-\right\| + \left\|\nu_{ij}^+ - \nu_{j+}^+\right\| \right\} \tag{43}$$

where $i = 1, 2, \dots, m$.

Eq. (44) also defines the separation measure degree between the IVIF negative-ideal solution and candidate A_i as follows:

$$d_i^- = \frac{1}{4m}\sum_{j=1}^{n}\left\{\left\|\mu_{ij}^- - \mu_{j^-}^-\right\| + \left\|\mu_{ij}^+ - \mu_{j^-}^+\right\| + \left\|v_{ij}^- - v_{j^-}^-\right\| + \left\|v_{ij}^+ - v_{j^-}^+\right\|\right\} \tag{44}$$

where $i = 1, 2, \ldots, m$.

Step 13: Experts' separation measures are combined using Eq. (45).

$$D_j^* = \sum_{k=1}^{k}\left(\lambda_k D_j^{*k}\right) \tag{45}$$

where λ_k is the weight of the decision maker k.

Step 14: Each alternative's proximity coefficient is calculated using Eq. (46).

$$C_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad \text{where} \quad 0 \le C_i \le 1. \tag{46}$$

Step 15: Alternatives ranking:
We may find the ranking order of all options in descending order of C_i's based on the relative closeness (C_i) (assessed in Eq. (46)).

5. Research guidance in academic and research institute

Research guidance is direction and assistance provided for enhancing each scholar's research skills and initiative. Accordingly, research scholars enable to reflect on their own work on their own methodologies and discover new things. For enhancement of research guidance, make researchers mindful of the motivation behind research work and drives to furnish them with the ability to pursue their own profession. Guidance is a fundamental component in all disciplines of the research field. Paper publication, conference support, and thesis support are the basic support of research scholars. Everyone wants to work on schedule, as in this fast-moving world, we want to finish everything on schedule to get the achievement. We finish our research work within the dispensed period and support our scholars all in their work through the research duration. In each second, we update ourselves and trust in our insight; we can give direction to different researchers to get an unprecedented idea that will give wonderful results. Since time is restricted for every research problem, so every researcher needs to approach and cooperate.

Compared to the academic institute, the research institute has more lab facilities and infrastructure. A research institute also has a restricted number of students under supervision. As a result, supervisors devote more time to their research work and the bonding level between professors and students is become very good and strong. However, professors at academic institutes are involved in many other things besides research. So they give relatively less time to scholars in their research. In academic and research institutes, there are various types of Ph.D. supervisors. They are distinguished by a number of criteria. Here we fixed the eight criteria for both academic and research institute supervisors, in which the beneficial criteria are Research area, Publication record, Collaboration record, Lab alumni status, Lab facility and the cost criteria are Academic position and administrative position. The best supervisor is selected based on these criteria, which are briefly discussed below.

5.1. *Research area* (C_1)

The research area is the most important thing in our Ph.D. career. Students always expect to do research in their own field. But sometimes, they do not get the actual supervisor who will do research on the topic of their choice. As a result, they are having a lot of difficulties with their Ph.D. work. It takes a long time to perceive this. If he is assigned a supervisor of his choosing, he will be able to grasp the topic in his own unique way. So, each of us should know the research area of the professor on which he is now working and discuss the topic with our supervisor before beginning our own research project. So we take this criteria as a beneficial.

5.2. *Publication record* (C_2)

We can also look at their research publication record, such as the number of published research articles and the quality of the journal in which their papers have been published when choosing a supervisor. Some have many research papers in low-quality journals, and some have few research articles, but in highly reputed journals like SCI, SCOPUS, and others also, many researchers have more national/international conference papers than journal papers. We can choose a Ph.D. supervisor by looking at all of these factors. It is also taken as a beneficial criterion.

5.3. *Collaboration record* (C_3)

Collaboration is extremely crucial in research work. If we discuss with our collaborators, We can come up with a good conclusion for our problem by contributing their thoughts. If any researcher has more collaborators, his research work will be better and easier to complete. We can also look at the collaborative history to find a supervisor and this is taken as a beneficial criterion.

Table 8
Data set collection description.

Variable	Type	Collection description
Research Area	Beneficial	Institute website, Personal web page
Publication Record	Beneficial	Institute website, Personal web page, Research gate, Google scholar, Scopus data base, DBLP
Collaboration Record	Beneficial	Institute website, Personal web page, Research gate, Google scholar, Scopus data base, DBLP
Academic Position	Cost	Institute website, Personal web page
Lab Alumni Status	Beneficial	Institute website, Personal web page
Lab Facility	Beneficial	Institute website, Personal web page
Administrative Position	Cost	Institute website, Personal web page
Academic and Experiences	Beneficial	Institute website, Personal web page

5.4. *Academic position* (C_4)

There may be different types of supervisors in academic professions such as assistant professor, associate professor, and professor. For Ph.D. work, Assistant professors can usually devote more time to students than Associate professors. The associate professor has more scholars than an assistant professor and is busy with various official activities, so they have some time to guide a student. Finally, professors have the highest number of research scholars and devote the least amount of time to their students. Also, they are very busy with various administrative work. However, they have the most research experience. Also, we have to pay attention to the age of the senior professor. If any professor retires during their Ph.D, then a supervisor has to be fixed anew. In that case, there may be a problem with the topic. So this criterion is taken as non-beneficial.

5.5. *Lab alumni status* (C_5)

To see the alumni status, we can simply comprehend how the supervisor guides the scholar, as well as how many students got a Ph.D. degree under the supervision of that guide and how long it takes to complete the course. It is also known about the scholar's current position or employment. This criterion is taken as beneficial criteria.

5.6. *Lab facility* (C_6)

It is essential to have a lab facility for Ph.D. work. Before entering Ph.D., we have to check all of the necessary equipment and facilities in the lab of the supervisor. So this criterion is considered as beneficial.

5.7. *Administrative position* (C_7)

Project guide or supervisor, who holds the position of Asso. Prof. or Prof. in an institution is generally mandated to hold and perform certain administrative positions and responsibilities. These administrative works cause a disruption in the guidance provided by them to the Ph.D scholars, as the former do need to devote their attention full time to the performance of these administrative tasks, thus somewhat hampering the guide-scholar relationship, which might affect the research activity afterwards. However, considering the aforementioned cons, we might take into consideration the pros also. Supervisors holding significant administrative ranks within the institution may provide certain leniency in terms of research apparatus availability, providing better research collaboration activities with other institutions, along with job opportunities within or outside the institution. Therefore this criterion is taken as non-beneficial.

5.8. *Academics and experiences* (C_8)

Along with the supervisor's research experience, it is also possible to find out where he received all of his academic degrees, including Ph.D. or Post-Doc. Sometimes, despite having a Ph.D. from a reputable institution and stellar academic credentials, a person may have minimal research experience. All of these things need to be kept in mind. Additionally, by looking at their profile, one can know how many awards and research grants the supervisor has received. Hence this criterion is considered as beneficial.

6. Extraction of research and administrative activities from different sources

There are several databases exist, from which we can know about researchers or Ph.D supervisors. See the below Table 8, then anyone can find the data related to some supervisor.

6.1. Publication databases

6.1.1. Google scholar

Google Scholar is the most momentous site good platform to know a lot of information about a particular researcher. When looking for a Ph.D supervisor, we should follow Google Scholar to become acquainted with the researcher's current research topic and the journal in which he or she published recently, as well as to gain an indication of the research quality by looking at the citations of each paper. The number of citations, h-index, and i-10 index indicate the quality of that research article. Here we can also see the author who cited our papers.

6.1.2. Researchgate

Another significant professional network for scientists and researchers is Researchgate. We easily find other researchers in our field and connect to each other by sharing our knowledge, skills, and information. We can also be aware of the supervisor's lab head and research group, which includes M.Sc., Ph.D, and Post-Doc students. We may learn about their research work over the last two to three years from their profile, including the number of papers, chapters, conferences, projects, and other publications.

6.1.3. Scopus/Scopus authors profile

Scopus is another main website to visit since it contains significant information about the performance of professors who are affiliated with ELSEVIER. We can see how well this person does in peer-reviewed conferences and articles, as well as how many citations, publication history, h-index they have, and everything else that is completely legitimate and will give the idea about the supervisor. So if we go to Scopus and search by author using the surname, initials of the author's first name, and possibly affiliation and institute, we should be able to track that author's performance.

6.1.4. Web of Science/Web of Science researcher id

The most advanced platform for searching and analyzing scientific citations worldwide is Web of Science (WoS), which is owned by Clarivate Analytics. The selection criteria for journals are more stringent and reliable than Scopus. The journals are evaluated by their impact factor, and the papers published by WoS indexed journals are of higher quality than those published by other journals. So the supervisors can be compared to see WoS id.

6.1.5. PubMed/PubMed id

In the medical field, PubMed is the database that is most frequently used. It could conduct sophisticated searches and filtering, link to full-text articles, and offer relevant article links. PubMed uses a PMID number as a unique identification. Here each article is indicated by PubMed id. We can find them by going to PubMed and writing the paper title, and seeing whether our supervisor is working in that field or not.

6.1.6. DBLP

DBLP is an online bibliography database that contains all of a researcher's publications and the type of journal in which they have published. As a result, there are a few steps to follow while analyzing any research on DBLP. Especially the electrical and computer science students check the steps, such as publication in IEEE Transactions, impact factor and publisher of that journal, Q ranking, ranking the conference where he/she is publishing, etc. Persons are identified in dblp in a variety of ways based on their name string. Persons are identified in DBLP in a variety of ways based on their name string. A unique identifier is also connected with a researcher as well as each publication record, journal, or conference paper.

6.1.7. RG-Score/Cite score

The RG Score and Cite score indicate how one's peers review his/her research work and its impacts. We believe that the best assessors of each other's work decide the quality of their work. The Researchgate score determines the supervisor's contribution to research and how many research articles are published. RG-Score is also influenced by their project work, as well as questions and answers. Also, The h-index is a simple approach to assess the influence of your research and that of others. It accomplishes this by examining a researcher's number of extremely good publications. The h-index is derived from the total number of articles published (N_p) and the number of citations (N_c) for each paper. To see the h-index, we can know how many citations the professor has. Those have more citations than some others; we can understand the research activity and level. The RG score and citation score can be used to evaluate a professor's research work and quality. These ratings can also be used to choose a supervisor.

6.2. CV from different websites

The curriculum vitae, often known as a CV or vita, is a detailed explanation of our educational history, teaching experience, research experience and publications, upper-level administrative positions in higher education, Professional association leadership positions, Research & consulting assignments, principal, or department chair. We usually look for a professor's CV on the institute's website. CVs from other websites, such as Live Career, Academia, Zety, Penn Career Service, and others, are also available.

Table 9
Ratting in linguistic terms for criteria and alternatives in crisp and IVIFNs.

Linguistic terms	Crisp number (0–9 scale)	Corresponding IVIF numbers Membership and non membership values in terms of intuitionistic fuzzy number
Extremely Important (EI)	9	([0.80, 0.90], [0.00, 0.10])
Strongly Important (SI)	7	([0.70, 0.80], [0.10, 0.20])
Important (I)	5	([0.60, 0.70], [0.20, 0.30])
Fairly Important (FI)	3	([0.50, 0.60], [0.30, 0.40])
Average Important (AI)	1	([0.50, 0.50], [0.50, 0.50])
Fairly Not Important (FNI)	1/3	([0.30, 0.40], [0.50, 0.60])
Not Important (NI)	1/5	([0.20, 0.30], [0.60, 0.70])
Strongly Not Important (SNI)	1/7	([0.10, 0.20], [0.70, 0.80])
Extremely Not Important (ENI)	1/9	([0.00, 0.10], [0.80, 0.90])

6.3. Institutions information

We can also look up a professor's profile on the institute's website to learn more about their research and experience. Also, we can become acquainted with the other scholars and how they work under that supervisor, and if we go to another department, we can learn about other topics such as whether or not the supervisor is in an administrative position, their behavior, communication skill and also about the department.

6.4. Other sites

There are other additional websites where we can have a rudimentary understanding of researcher details. Linkedin, People search, Google Groups, TruePeopleSearch, Intelius, and more sites are among them. We may also get information about supervisors from other social media platforms.

7. Data collection and numerical illustration

This section discourses on the data sources and collection procedure in detail. Also, the numerical computations of the supervisor selection process are shown in this section.

7.1. Data source for the study

This section describes the data sources of this research. Table 8 lists the several ways in which we collected data about supervisor criteria from different sources during the study. There are two categories of criteria: beneficial criteria and non-beneficial (cost) criteria. This table describes the types, sources, and data for all criteria in detail.

7.2. Linguistic terms expressed in IVIF numbers in different scale

We solve this problem by taking both Crisp and IVIF numbers of the linguistic variable that are displayed in Table 9.

7.3. Comparison of criteria to criteria carried out by DM

We have employed two distinct decision-making models to offer clarification using three DMs. We first prove the best supervisor with one Decision-maker (DM), and then we prove it with three DMs.

7.3.1. Supervisor selection from single DM prospective(as Ph.D. seeking student)

A student looking for a supervisor should rate the criteria according to that supervisor. He assigns the rating of criteria to criteria by linguistic variable which is displayed in Table 10 and the IVIF AHP methodology is used to determine the criteria weights which is shown in Table 11.

From Table 11, for the single decision maker viewpoint using IVIF-AHP methodology, the criteria weight are as follows $C_1 > C_2 > C_8 > C_3 > C_5 > C_6 > C_4 > C_7$.

Table 10
Comparison Matrix between criteria to criteria by Ph.D. seeking student.

Criteria	Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab Alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Research Area C_1	AI	I	FNI	EI	FI	SI	EI	FI
Publication Record C_2	NI	AI	I	SNI	FI	SI	I	EI
Collaboration Record C_3	FI	NI	AI	FI	EI	NI	SI	FI
Academic Position C_4	ENI	SI	FNI	AI	FNI	SNI	I	NI
Lab Alumni Status C_5	FNI	FNI	ENI	FI	AI	FI	EI	SNI
Lab Facility C_6	SNI	SNI	I	SI	FNI	AI	SI	NI
Administrative Position C_7	ENI	NI	SNI	NI	ENI	SNI	AI	ENI
Academics & Experiences C_8	FNI	ENI	FNI	I	SI	I	EI	AI

Table 11
Representation of the criteria weight for single DM prospective using IVIF AHP.

Criteria	Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Weight	0.1707	0.1500	0.1428	0.0908	0.1214	0.1178	0.0625	0.1440

Table 12
Comparison Matrix between criteria to criteria by a person who known the student, work in the institute.

Criteria	Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Research Area C_1	AI	I	NI	SI	FI	SI	EI	SI
Publication Record C_2	NI	AI	I	NI	FI	EI	EI	I
Collaboration Record C_3	I	NI	AI	FI	SI	SNI	I	SI
Academic Position C_4	SNI	I	FNI	AI	NI	ENI	EI	FNI
Lab Alumni Status C_5	FNI	FNI	SNI	I	AI	I	FI	FNI
Lab Facility C_6	SNI	ENI	SI	EI	NI	AI	I	SNI
Administrative Position C_7	ENI	ENI	NI	ENI	FNI	NI	AI	ENI
Academics & Experiences C_8	SNI	NI	SNI	FI	FI	SI	EI	AI

Table 13
Comparison Matrix between criteria to criteria by an academician from other institute, known to student.

Criteria	Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Research Area C_1	AI	SI	NI	SI	I	EI	EI	I
Publication Record C_2	SNI	AI	I	FNI	I	SI	EI	FI
Collaboration Record C_3	I	NI	AI	AI	EI	FNI	FI	EI
Academic Position C_4	SNI	FI	AI	AI	FNI	NI	SI	SNI
Lab Alumni Status C_5	NI	NI	ENI	FI	AI	I	I	SNI
Lab Facility C_6	ENI	SNI	FI	I	NI	AI	I	NI
Administrative Position C_7	ENI	ENI	FNI	SNI	NI	NI	AI	SNI
Academics & Experiences C_8	NI	FNI	ENI	SI	SI	I	SI	AI

7.3.2. Supervisor selection from multiple DMs viewpoint, here consider three DMs

(a). Ph.D. seeking student

Student who looking for a supervisor should rate the criteria according to that supervisor. He assigns the rating of criteria to criteria by linguistic variable which is displayed in Table 10.

(b). A person known to the student, work in the institute

Now we select another person from other department of same institute as a DM to get the rating of criteria about the supervisor. Table 12 demonstrates how that person rated the various factors.

(c). An academician from other institute, known to student

The final expert is chosen from a different institute who is familiar with all criteria to select a supervisor who guide Ph.D. students and we are asking the rate of criteria from him, which is displayed in Table 13. Finally we get the aggregating result of criteria weight in Table 14 from the Tables 10, 12 & 13 by the Eq. (32) using IVIF AHP.

Therefore, from Table 14 for three DMs opinions using IVIF AHP technique, the order of criteria in proportion to the weight is $C_1 > C_2 > C_3 > C_8 > C_5 > C_6 > C_4 > C_7$.

Table 14
Representation of the criteria weight for multiple DMs prospective using IVIF AHP.

Criteria	Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Weight	0.1732	0.1564	0.1485	0.1026	0.1135	0.1108	0.0625	0.1325

Table 15
Comparison table in linguistic variables by Ph.D. seeking student between Criteria and Alternatives.

DM1		Criteria							
		Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Alternative	Supervisor A	EI	FNI	I	FI	AI	SNI	SI	AI
	Supervisor B	I	FI	NI	EI	AI	SI	ENI	I
	Supervisor C	FI	EI	I	NI	SI	EI	ENI	AI
	Supervisor D	AI	SI	I	FI	EI	FNI	AI	I
	Supervisor E	EI	NI	SI	FI	AI	SNI	EI	I

Table 16
Relative closeness between alternatives and ranked them using TOPSIS methodology.

Alternatives	d_i^+	d_i^-	$C_i = \frac{d_i^-}{d_i^+ + d_i^-}$	Ranking
Supervisor A	0.271247892	0.206461619	0.432190723	3
Supervisor B	0.335936935	0.141772576	0.296775703	5
Supervisor C	0.135368103	0.342341408	0.716630923	1
Supervisor D	0.215522693	0.262186817	0.548841527	2
Supervisor E	0.300228473	0.177481038	0.371525025	4

Table 17
Compression table between criteria and alternatives in linguistic terms by DMs.

DM2		Criteria							
		Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Alternative	Supervisor A	SI	NI	SI	AI	FNI	ENI	EI	AI
	Supervisor B	I	FI	NI	EI	AI	I	SNI	I
	Supervisor C	I	EI	FI	SNI	SI	EI	ENI	AI
	Supervisor D	FI	SI	I	FI	SI	FNI	AI	I
	Supervisor E	EI	NI	SI	FI	AI	ENI	EI	I

7.4. Criteria to alternatives comparison matrix conducted by DM in linguistic terms

7.4.1. Supervisor selection from single DM prospective (as Ph.D. seeking student)

A single student from a specific department at a reputable university evaluated the five instructors in Table 15 and selected the top supervisor based on eight factors and used IVIF TOPSIS to determine the supervisor's ranking, as indicated in Table 16.

7.4.2. Supervisor selection from multiple DMs prospective, here consider three DMs

(a). Ph.D. seeking student

One student is taken from Mathematics department as a first DM, give opinion of the five supervisor based on those eight criteria shown in Table 15.

(b). A person known to the student, work in the institute

We have taken another professor of Physics department as a second DM to know about all professor of Maths department and rated in Table 17.

(c). An academician from other institute, known to student

Lastly, we take information about the professor of the Mathematics department from an academician who is belonging to another reputed institute, the five supervisors which is described in Table 18.

Finally, the ranking of the supervisor as alternative using IVIF TOPSIS shown in the Table 19 by the given rating three DMs in Tables 15 and 17 & 18.

Table 18
Comparison table between criteria and alternatives in linguistic terms by DMs.

DM3		Criteria							
		Research area (C_1)	Publication record (C_2)	Collaboration record (C_3)	Academic position (C_4)	Lab Alumni status (C_5)	Lab facility (C_6)	Administrative position (C_7)	Academics and experiences (C_8)
Alternative	Supervisor A	EI	NI	I	AI	FNI	SNI	EI	FI
	Supervisor B	FI	I	FNI	SI	FNI	I	SNI	I
	Supervisor C	FI	SI	FI	SNI	EI	SI	SNI	AI
	Supervisor D	AI	EI	FI	AI	EI	NI	FI	FI
	Supervisor E	SI	FNI	I	AI	AI	SNI	SI	FI

Table 19
Relative distances between alternatives and ranked them applying TOPSIS technique.

Alternatives	d_i^+	d_i^-	$C_i = \frac{d_i^-}{d_i^+ + d_i^-}$	Ranking
Supervisor A	0.303981108	0.154267333	0.336645626	4
Supervisor B	0.328351402	0.129897039	0.283464224	5
Supervisor C	0.086162174	0.372086267	0.811974976	1
Supervisor D	0.168576440	0.289672001	0.632128722	2
Supervisor E	0.291134509	0.167113932	0.364679760	3

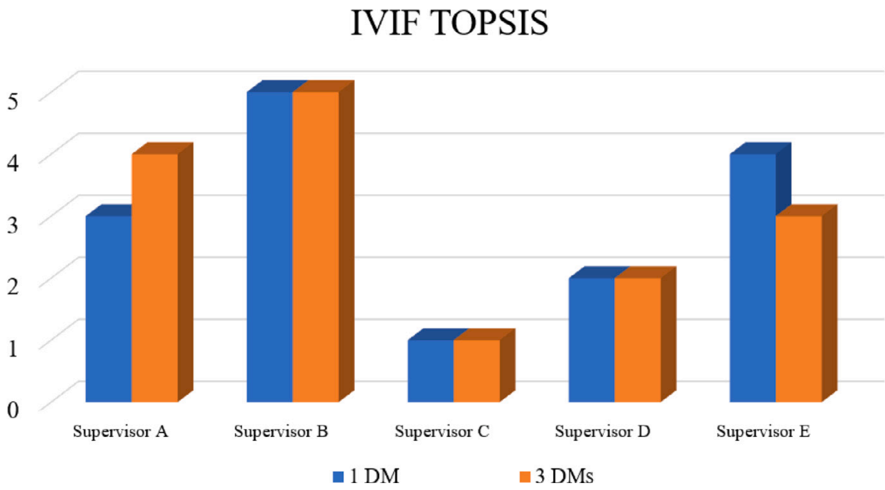


Fig. 9. Ranking obtained by the two different type of DMs prospective by TOPSIS method.

Remarks 1. From Tables 16, 19 portrayed Fig. 9. From Fig. 9 we conclude that the Supervisor **C** is the most beneficial guide as per 1DM and 3DMs preferences, 2nd optimal supervisor is given as Supervisor **D** by both DM groups. 3rd and 4th position goes to Supervisor **A** and Supervisor **E** respectively for 1DM where in 3DMs the reversed order. Lastly, Supervisor **B** is the last choice for both the DMs group.

8. Sensitivity analysis

Sensitivity analysis is conducted by interchanging some beneficial and non-beneficial criteria or removing some criteria for both cases of single and multiple DMs. We considered four different cases. Tables 20, 21, 22 & 23 shows rank of the alternatives and Figs. 9, 10, 11 & 12 give their graphical representation determined for each of four cases based on 1DM and 3DMs respectively using IVIF TOPSIS. The four cases are described below.

8.1. Removing the criteria lab alumni status (C_5)

Sometimes students are unaware of the lab alumni status of supervisor and they join PhD based on their preferred area. So we remove this criteria but rest criteria are unchanged. As a result the rank of Supervisor B & Supervisor C are same but others supervisors' rank are changed for both single and multiple DMs respectively. Table 20 and Fig. 9 illustrate how the rank of supervisor is changed for one DM and three DMs.

Table 20
Ranking between one DM and three DMs using MCDM method TOPSIS by removing the criteria Lab Alumni Status (C_5).

Alternatives	One DM	Three DMs
Supervisor A	2	4
Supervisor B	5	5
Supervisor C	1	1
Supervisor D	3	2
Supervisor E	4	3

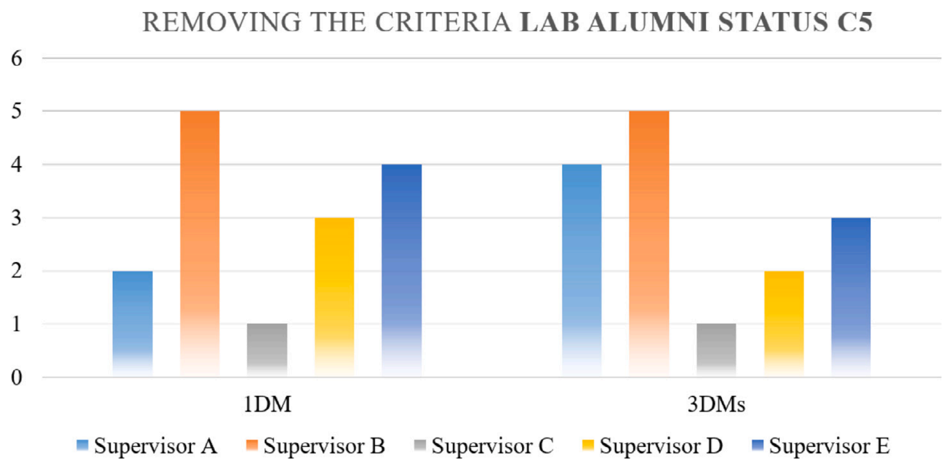


Fig. 10. Sensitivity analysis by removing the criteria Lab Alumni Status (C_5) using TOPSIS method.

Table 21
Ranking between one DM and three DMs using MCDM technique TOPSIS by removing the criteria Lab Facility (C_6).

Alternatives	One DM	Three DMs
Supervisor A	3	4
Supervisor B	5	5
Supervisor C	1	1
Supervisor D	2	2
Supervisor E	4	3

Remarks 2. From Table 20 & Fig. 10, we see that the Supervisor **C** is the most suitable guide as per 1DM and 3DMs preferences, 2nd preferences given as Supervisor **B** by both DM group. 3rd position is little bit complex, Supervisor **D** becomes 1DM choice whereas Supervisor **E** becomes 3DM choice. 4th position is also complex here. Supervisor **A** becomes this position for 3 DM choice whereas Supervisor **E** becomes 1DM choice. Supervisor **B** is placed the last choice for both DM group.

8.2. Removing the criteria lab facility (C_6)

There is no need to use of lab for theoretical subjects. Consequently, someone may not use this criterion to choose a supervisor. So in this case, if we eliminate this criterion, the three supervisors B, C, and D have the same rank and the other two are changed for one DM and three DMs respectively, which is shown in Table 21 and Fig. 11.

Remarks 3. From Table 21 & Fig. 11, we see that the Supervisor **C** is the most suitable guide as per 1DM and 3DM preferences, 2nd preferences given as Supervisor **D** by both DM group. 3rd position is little bit complex, Supervisor **A** becomes 1DM choice whereas Supervisor **E** becomes 3DM choice. 4th position is also complex here. Supervisor **A** becomes this position for 3 DM choice whereas Supervisor **E** becomes 1DM choice. Supervisor **B** is placed the last choice for both DM group.

8.3. Removing the criteria administrative position (C_7)

Generally, the professors who hold administrative positions, may unable to devote as much time to their scholars. If someone may enter a PhD programme with self confidence, they can do it whether their supervisor is supportive or not. Then in that case if we remove this criteria, there is no impact of the rank of the supervisor to that scholar, shown in Table 22 & Fig. 12.

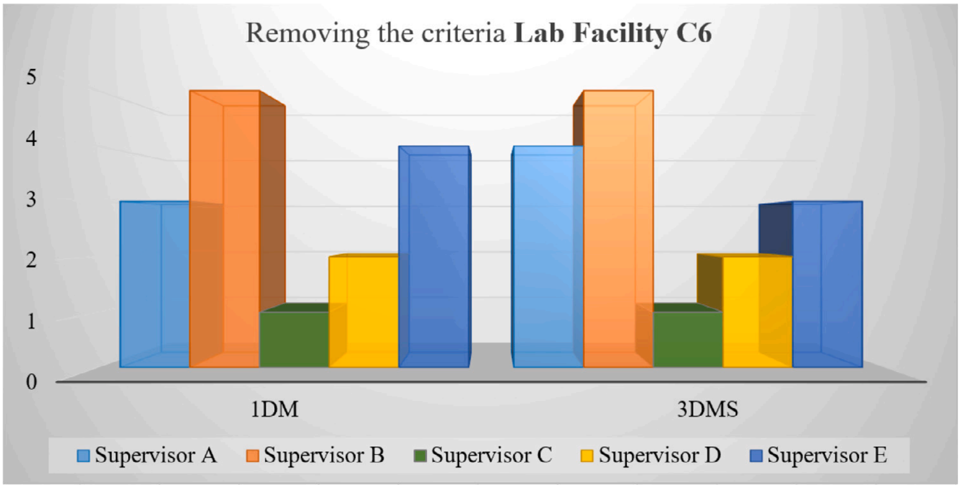


Fig. 11. Sensitivity analysis by removing the criteria Lab Facility (C_6) using TOPSIS method.

Table 22
Ranking between one DM and three DMs using MCDM technique TOPSIS by removing the criteria Administrative Position (C_7).

Alternatives	One DM	Three DMs
Supervisor A	4	4
Supervisor B	5	5
Supervisor C	1	1
Supervisor D	2	2
Supervisor E	3	3

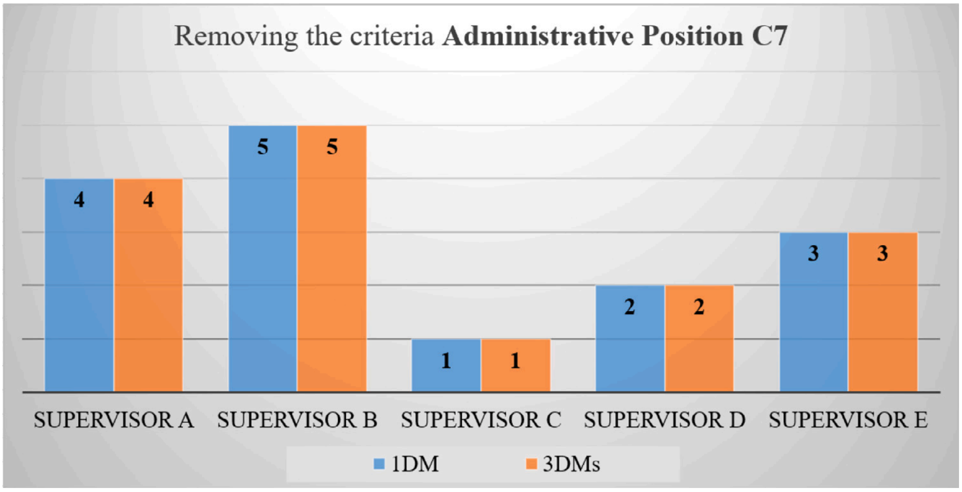


Fig. 12. Sensitivity analysis by removing the criteria Administrative Position (C_7) applying TOPSIS technique.

Remarks 4. From Table 22 & Fig. 12, we see that the Supervisor **C** is the most suitable guide as per 1DM and 3DM preferences, 2nd, 3rd, 4th & 5th preferences given as Supervisor **D**, Supervisor **E**, Supervisor **A** & Supervisor **B** respectively, by both DM group.

8.4. Convert cost criteria (C) to benefit criteria (B)

There are two cost criteria, which are Academic position and Administrative position. Sometimes those who are in academic or administrative position, they can assist the students for official work and/or job placement also. So if we take this two criteria as a beneficial, then supervisor B & C have no change on their rank but others are changed in Table 23 & Fig. 13.

Table 23
Ranking between one DM and three DMs using MCDM technique TOPSIS by interchange Cost criteria (C) to Benefit criteria (B).

Alternatives	One DM	Three DMs
Supervisor A	5	4
Supervisor B	4	5
Supervisor C	1	1
Supervisor D	3	2
Supervisor E	2	3

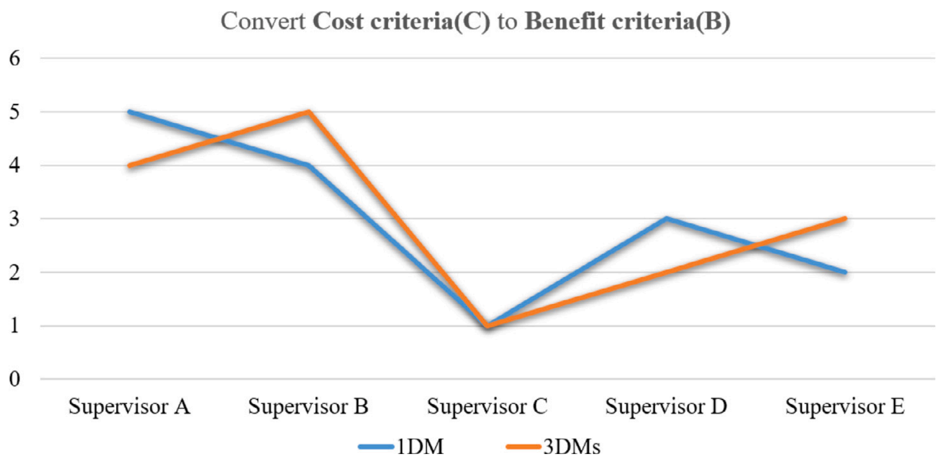


Fig. 13. Sensitivity analysis by interchanging Cost criteria (C) to Benefit criteria (B) using TOPSIS method.

Table 24
Ranking between Crisp number, Fuzzy number, Intuitionistic fuzzy number and **IVIF** using MCDM technique TOPSIS on the bases of one DM.

Alternatives	Crisp number	Fuzzy number	Intuitionistic fuzzy number	IVIF
Supervisor A	2	5	5	**3**
Supervisor B	5	4	3	**5**
Supervisor C	1	1	1	**1**
Supervisor D	4	3	2	**2**
Supervisor E	3	2	4	**4**

Remarks 5. From Table 23 & Fig. 13, we see that the Supervisor **C** is the most suitable guide as per 1DM and 3DM preferences, 2nd preferences given as Supervisor **E** & Supervisor **D** by 1DM & 3DM group respectively. 3rd position is little bit complex, Supervisor **D** becomes 1DM choice whereas Supervisor **E** becomes 3DM choice. 4th position is also complex here, Supervisor **A** becomes this position for 3 DM choice whereas Supervisor **B** becomes 1DM choice. Supervisor **A** & Supervisor **B** is placed the last choice for 1DM & 3DM group respectively.

9. Comparative analysis

9.1. Comparative analysis between crisp number, fuzzy number & intuitionistic fuzzy number (IF)

To compare the results, four different type of fuzzy numbers are used for both of single and three DMs. We start by using a crisp number to determine the rank of the supervisor before switching to a fuzzy number, which is more precise. Then we obtain the rank by intuitionistic fuzzy number, where both the membership and non-membership function are used to determine the rank of supervisor and perhaps yielding a more accurate result than a fuzzy number. Finally the membership and non-membership are expressed in interval numbers, thus we may achieve a more accurate result using this. Ultimately, we obtained a result using an interval valued intuitionistic fuzzy number. Considering each cases we may say that the supervisor C is the best among them. Tables 24 & 25 and Figs. 14 & 15 display the variation in supervisor rank for single and three DMs.

Remarks 6. In case of single DM describe in Table 24 and Fig. 14, Supervisor **C** is the most preferable in IVIF-TOPSIS, Crisp-TOPSIS, Fuzzy TOPSIS & IF-TOPSIS respectively. But for other position, there are different supervisor based of different fuzzy environment like for IVIF-TOPSIS; Supervisor **D** is in 2nd, Supervisor **A** is in 3rd, Supervisor **E** is in 4th and Supervisor **B** is in 5th position

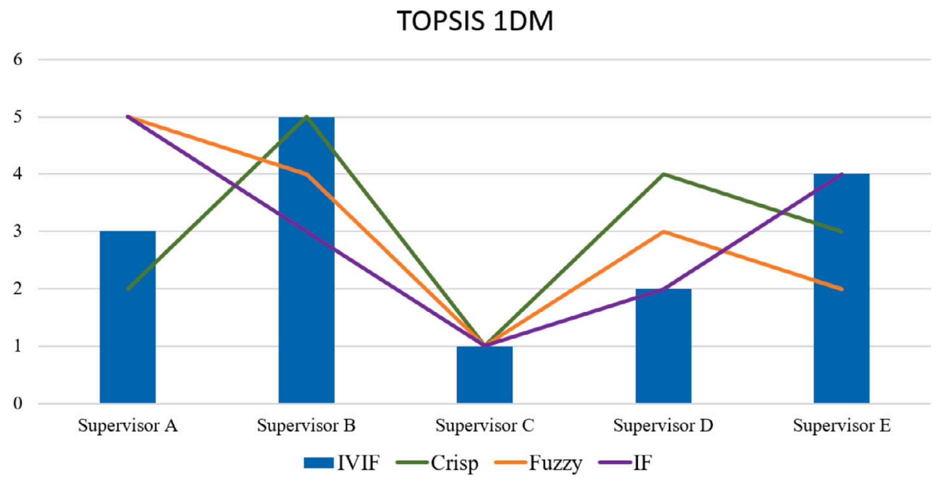


Fig. 14. Comparative analysis between Crisp number, Fuzzy number, Intuitionistic fuzzy number and IVIFN.

Table 25
Ranking between Crisp number, Fuzzy number, Intuitionistic fuzzy number and **IVIF** using MCDM technique TOPSIS on the bases of three DMs.

Alternatives	Crisp number	Fuzzy number	Intuitionistic fuzzy number	IVIF
Supervisor A	3	4	5	**4**
Supervisor B	5	5	3	**5**
Supervisor C	1	1	1	**1**
Supervisor D	4	2	2	**2**
Supervisor E	2	3	4	**3**

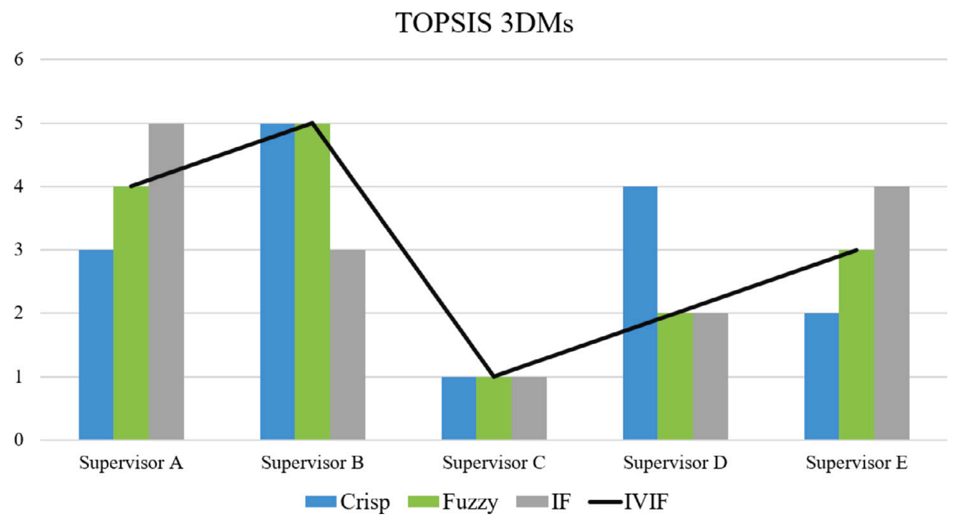


Fig. 15. Comparative analysis between Crisp number, Fuzzy number, Intuitionistic fuzzy number and IVIFN.

respectively. For Crisp-TOPSIS; Supervisor **A** is in 2nd, Supervisor **E** is in 3rd, Supervisor **D** is in 4th, Supervisor **B** is in 5th position respectively. For fuzzy TOPSIS; Supervisor **E** is in 2nd, Supervisor **D** is in 3rd, Supervisor **B** is in 4th & Supervisor **A** is in 5th position respectively. For IF-TOPSIS; Supervisor **D** is in 2nd, Supervisor **B** is in 3rd position, Supervisor **E** is in 4th position, Supervisor **A** is in 5th position respectively.

Remarks 7. In case of 3DM, Table 25 and Fig. 15, Supervisor **C** is the best among five supervisors for all case of different fuzzy MCDM methodology. Then the 2nd choice is Supervisor **D** by Fuzzy, IF & IVIF TOPSIS technique and Supervisor **E** by Crisp TOPSIS approach. The 3rd choice is Supervisor **E** by Fuzzy and IVIF TOPSIS & Supervisor **A** by Crisp TOPSIS & Supervisor **B** by IF-TOPSIS.

Then the 4th position are Supervisor **A** by Fuzzy and IVIF TOPSIS, Supervisor **D** by Crisp, Supervisor **E** by IF TOPSIS. Finally the last position is Supervisor **B** by Crisp, Fuzzy & IVIF TOPSIS, and Supervisor **A** by IF TOPSIS technique.

10. Conclusion of this study

Selecting a Ph.D. supervisor is a significant step for students entering the research field. How a student chooses the guide in a correct scientific manner, is quite difficult. In this present study, we proposed a methodology for supervisor selection that is based on some collective data sets under the guidance of some decision-makers. Our evaluation method is very much straight forwards than previously published work-inspired supervisor selection problems.

The whole work is done under the MCDM methodology, namely AHP coupled TOPSIS method in an interval-valued intuitionistic fuzzy environment. First, we collect a few important criteria and collect useful data from three types of experts: students, persons working in the institute, who are known to the scholars and academicians from other institutes. Using the interval-valued intuitionistic fuzzy AHP, firstly the weights of the criteria have been calculated and then, the supervisors have been ranked by interval-valued intuitionistic fuzzy TOPSIS. In criteria weight evaluated by AHP, the research area (C_1) gets maximum weight followed by publication record (C_2) occupied second position, and so on. Similarly, for ranking alternatives using TOPSIS methodology, Supervisor C is the optimal option followed by Supervisor D is the second option, and so on. Moreover, sensitivity analysis and comparative study are also carried out to check the stability and better analysis of the proposed hybrid approach.

10.1. Limitation of the study

An interval-valued intuitionistic fuzzy number is utilized to deal with uncertainty and improve imprecision in evaluating criteria and potential supervisors. To solve this type of problem, number of criteria and alternatives are chosen as decision maker choices. All criteria may be taken as beneficial or cost or both, and it can be quantitative or qualitative. The score and accuracy function is used only for measuring IVIF numbers and ranking them by score value. Depending on the criteria a candidate uses, the rank of the supervisor can differ. In many cases, for an unknown candidate, some supervisor does not want to interact leniently. Consequently, it is difficult to compare other supervisors' performance on many of the criteria objectively. In IFS, any uncertain situation may not be measured by single real valued membership and non membership function. Also in case of IVIF set, we cannot get the exact belongingness of an element. When the number of criteria and alternatives are increased, then to build the AHP model, it requires much more time and effort. The consistency of weight in AHP may not be derived for different fuzzy numbers. The ranking of alternatives in TOPSIS, however, can vary depending on the fuzzy environment.

10.2. Future research scope

In future research, we can take more criteria and associated sub-criteria. Also, we can take several decision-maker opinions and study the corresponding results. All the analyses support the efficiency of the proposed approach. Therefore, it may be concluded that the framework suggested in the present study may be utilized in different decision making problems which involve various levels of uncertainties. Also, multiple MCDM methodologies shall be applied to check the accuracy of the system. More cases shall be conducted in sensitivity analysis and comparative analysis to analyze the stability of the system.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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