A vertical take-off and landing (VTOL) unmanned aircraft vehicle trajectory control model with air-launched missiles

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Abstract

Purpose – In today's technology, the significance of unmanned aerial vehicles is steadily increasing. Many unmanned aerial vehicles design, especially those used for military purposes, have achieved autonomy from human operators. Undoubtedly, one of the most crucial features of these aircraft is their vertical take-off and landing (VTOL) capability. Inspired by quadrotor methodology, this paper aims to conduct, a modeling of an aircraft with VTOL capabilities.

Design/methodology/approach – The impact of releasing air-launched missiles, considered as the useful payload carried during the flight of the aircraft, has been taken into account in this modeling. The release of air-launched missiles disrupts both the symmetric structure of the system and alters the mass and inertia parameters. Simulations were conducted to investigate scenarios involving the simultaneous release of all air-launched missiles and their release at different times.

Findings – The investigation focused on determining how quickly an aircraft, aiming to consecutively hit targets, can return to its desired trajectory. The time interval between the consecutive releases of two air-launched missiles has been identified.

Originality/value – It is crucial for a VTOL-capable aircraft to possess a unique modeling structure to examine its capability of releasing air-launched missiles in various scenarios. This entails understanding not only the aircraft's VTOL functionality but also its ability to effectively release missiles in different operational conditions.

Keywords Trajectory control, Vertical take-off, Vertical landing, Air-launched missile, Unsymmetric structure

Paper type Research paper

1. Introduction

As a result of technological advancements, the designs of aircraft are becoming widespread in every aspect of our lives and are being used for many different tasks. There are many aircraft projects being worked on by different nations, ranging from amateur to professional levels (Goraj, 2014). When examining these studies in the field of aviation, there are projects aimed at passenger transport (Goraj *et al.*, 2019), emergency medical service (Goetzendorf-Grabowski *et al.*, 2021) as well as the development of tracking and detection systems with unmanned aircraft vehicle (UAV) for search and rescue operations (Frąckowiak and Goraj, 2023). The importance of UAV designs has increased, especially in the defense industry, with the recent advancements in technology. The subject of this study encompasses an aircraft system with a vertical take-off and landing (VTOL) capability. While a traditional fixed-wing aircraft typically has a single propulsion engine, the addition of four extra motors to the system provides it with VTOL capability. Therefore, combining the advantages of a rotary-wing aircraft with those of a fixed-wing aircraft forms the concept of a VTOL UAV. The concept of being able to perform vertical takeoff and transition to horizontal flight has been explored by various nations in many military applications (Zhou *et al.*, 2020b). Such aircraft can be operational without the need for conventional runways or air bases. This feature can offer advantages such as providing flexibility in military operations, being able to take off and land quickly from narrow areas or participating in operations surprisingly. VTOL technology is expected to be incorporated not only in military applications but also in flying cars, which are considered as the future of mobile transportation (Pan and Alouini, 2021). The flying car concept will provide the advantage of vertical takeoff and landing without the need for traditional airports or helicopter pads, making them suitable for movement in urban areas and congested regions (Archdeacon *et al.*, 2020).

VTOL capability has been studied in various designs by different research groups, both in large and small scales. Such designs are being developed and researched for a wide range of applications. A research group from Poland have worked on new design of aircraft with four-rotor, in which two of the four rotors are mounted on the nacelle capable of rotation

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(Czyba *et al.*, 2018). The thrust engines in their design are capable of both vertical and horizontal flight. Giuseppe and John in their research, aimed to investigate the maneuvering capabilities of tilt-rotor aircraft. The primary task they outline is modeling and investigating various dynamic and aerodynamic regimes (Notarstefano and Hauser, 2010). A research group from Istanbul Technical University worked on the fixed-wing VTOL system. They presented a nonlinear mathematical model including hover, forward flight, and transition regimes (Yuksek *et al.*, 2016). As a different study, annular wing design instead of classical fixed wing is also a research topic. The annular wing used here enhances human safety by enshrouding the propeller blades (Gill and D'Andrea, 2020). There is also a flying wing structure have also available that researchers are working on, that is capable of both traditional flight like a normal fixed-wing aircraft and vertical hovering like a multi-copter (Bliamis *et al.*, 2021). In another flying wing concept, a design based on flight test data of an autopilot-loaded UAV aircraft and the development methodology of a multidisciplinary optimization framework has been presented by a research group from Poland (Mieloszyk *et al.*, 2020). Researchers from Japan presented a concept of a quad tilt-wing UAV, which takes off and lands vertically and cruises similarly to a fixed-wing airplane. They have proved their proposed design by flight tests using a small proof-of-concept model (Muraoka *et al.*, 2009). Different studies and implementations have also been proposed in the literature for VTOL UAV systems to achieve the advantage of a multirotor UAV while having the ability to travel fast to reach a further distance (Lyu *et al.*, 2017; Peloquin *et al.*, 2017; Abd Rahman *et al.*, 2018; Zhou *et al.*, 2020a; Kumar *et al.*, 2021).

Up to this point, as seen in the examined studies, VTOL UAV systems have been the subject of research in various structures and scales. In addition to these, the trajectory control problem of an aircraft, deriving its dynamic model, and studying the impact of changing system parameters on the trajectory during flight are also significant issues. The dynamics of a quadrotor largely reflect those of VTOL UAV systems. This similarity is leveraged in research efforts to develop control laws specifically designed for multi-rotor systems, which inherently tend to be unstable (Chodnicki *et al.*, 2019). A control strategy to stabilize the position of a VTOL UAV in crosswind despite unknown aerodynamic effects is investigated in Pflimlin *et al.* (2007). The mathematical model of the VTOL aircraft system with the perspective of aircraft forces and moments was investigated for six degrees of freedom model in Zhang *et al.* (2017). Avionics and control system of VTOL UAV was presented to show control scenario of proposed UAV in Hadi *et al.* (2016). Zou and Meng have investigated the coordinated trajectory tracking problem of multiple VTOL UAVs in the case of unidirectional information flow is assumed (Zou and Meng, 2019). Onen *et al.* have designed and presented linear quadratic regulators and linear quadratic tracking controllers for the attitude control of the aircraft in vertical takeoff and hover flight conditions (Onen *et al.*, 2015). Roberts and Tayebi have worked on an adaptive position-tracking control scheme for VTOL UAVs under a set of bounded external disturbances (Roberts and Tayebi, 2010). Sivrioglu has developed a mathematical model for short take-off and vertical landing (STOVL) aircraft dynamics a model system that resembles such aircraft dynamics. The proposed model system have used the thrust of axial fans to hover the whole system in the air (Sivrioglu, 2023a, 2023b).

In this study, a fixed-wing UAV system with VTOL capability, which can be used for military purposes, has been investigated. The considered system, in vertical takeoff mode, actually shares a similar dynamic model with a quadrotor. The dynamic model assumes that air vehicles, like quadrotors, have a symmetric structure in the $-xz$ and $-yz$ planes (Alderete, 1995). As seen in Figure 1, the VTOL UAV system, assumed to be symmetric only in the $-xz$ plane, when has right and left missiles. If one of them is released during flight, the symmetry of the system will be disrupted, and the inertia tensor and total mass will change. Therefore, the dynamic model under consideration should be implemented to account for changes in the system's inertia tensor and total system mass. This study examines a scenario involving a UAV performing vertical takeoff in Quadrotor mode and following a specific trajectory. During the flight, the UAV sequentially releases missiles from the right and left sides at different times.

The remaining part of the study is structured as follows: In Section 2, the UAV system structure is presented. In Sections 3 and 4, the mathematical model of quadrotor and VTOL UAV system is given respectively. The subsequent section addresses the trajectory control problem for an aircraft flying in quadrotor mode. A control design is presented for the case where air-missiles are released at different times during the flight.

2. Unmanned aircraft vehicle model structure

The structure of aircraft is schematically shown in Figure 1(a). The aircraft model has four propellers for vertical take-off and a single propulsion propeller for forward motion. While vertical take-off engines are designed as electric motors, forward thrust engine is designed as fuel engine. In operation, the aircraft takes off with electric motors, then switches to cruise flight mode and only the fuel engine is active.

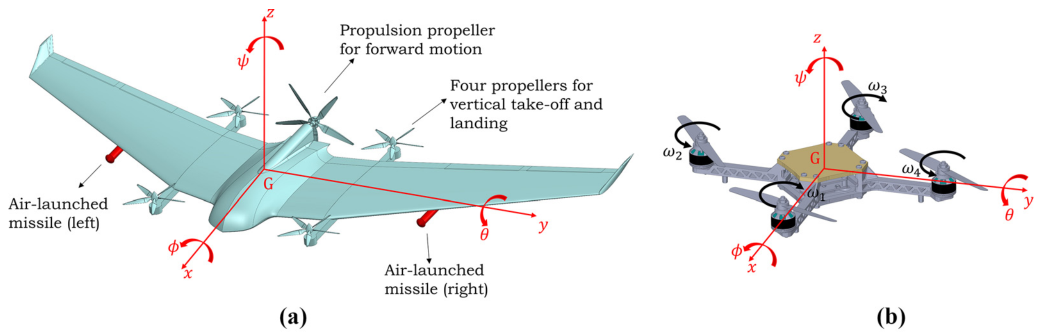
It is assumed that the aircraft is rigid and mass center point G is known. In each wing, an identical air-launched lightweight missile is attached to an equal distance from the mass center. A local coordinate system is assumed at the center of the mass to define the translational and rotational motions. The roll, pitch and yaw motions around x, y and z axes are depicted with angular variables ϕ, θ and ψ, respectively.

In this unmanned aerial vehicle system, the launch of the missile can be handled for two different situations. The first is the launch of the missile when the aircraft reaches a certain height H with the vertical take-off propellers. In this situation, we assume that vertical lift propellers are in operation during launching. The other situation can be considered as the launch of the missile during the forward movement of the aircraft.

3. Model of a quadrotor

The quadrotor in Figure 1(b) refers to a four-rotor design. This type of aircraft is a rotary wing aircraft with four propellers. This allows the quadrotor to move in the desired direction by adjusting the speed of each rotor independently. Quadrotors are the most commonly known VTOL UAVs. A local coordinate system of the quadrotor is assumed at the center of the mass to define the translational and rotational motions. The roll, pitch and yaw motions around x, y and z axes are depicted

Figure 1 (a) VTOL Unmanned aircraft vehicle model and (b) quadrotor model



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with angular variables ϕ, θ and ψ, respectively. Note that, due to its unique design, a quadrotor is assumed to have a symmetric structure in the $-xz$ and $-yz$ planes. Quadrotor rotation matrix from the local coordinate system to the inertial frame can be defined as follows:

$$\boldsymbol{R} = \begin{bmatrix} C_\theta C_\psi & S_\theta C_\psi S_\phi - S_\psi C_\phi & S_\theta C_\psi C_\phi + S_\psi S_\phi \\ C_\theta S_\psi & S_\theta S_\psi S_\phi + C_\psi C_\phi & S_\theta S_\psi C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \quad (1)$$

where $C_{\{\cdot\}}$ represents the cosine function and, $S_{\{\cdot\}}$ represents the sine function. The inertia tensor of a quadrotor can be defined as follows due to its symmetric structure:

$$\boldsymbol{I}_{quad} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (2)$$

As evident from the inertia tensor, all terms except the diagonal ones are zero for a quadrotor. The dynamic equations of a quadrotor can be separately considered for translational and rotational motion. Equation (3) provides the motion equation for the translational dynamics of the quadrotor:

$$m\ddot{\xi} + mg\boldsymbol{i}_{\mathbf{z}} = \boldsymbol{T} \quad (3)$$

Here, m represents the total mass of the quadrotor, assumed to remain constant during flight, $\xi = [x\, y\, z]^T$ is the absolute linear position of the quadrotor, $\boldsymbol{i}_z = [0\ \ 0\ \ 1]^T$ is the gravity compensation vector and $\boldsymbol{T}$ is the thrust vector. The transformation matrix for angular velocities from the inertial frame to the body frame can be defined as follows:

$$\boldsymbol{W}_{\boldsymbol{\eta}} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix} \quad (4)$$

The Jacobian matrix of quadrotor can be defined by using the transformation matrix from the inertial frame to the body frame as follows:

$$\boldsymbol{J} = \boldsymbol{W}_{\boldsymbol{\eta}}^{\boldsymbol{T}} \boldsymbol{I}_{quad} \boldsymbol{W}_{\boldsymbol{\eta}} \quad (5)$$

The dynamic equation for quadrotor rotational dynamics is as follows:

$$\boldsymbol{J\eta} + \boldsymbol{C}(\boldsymbol{\eta},\dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} = \boldsymbol{\tau}_{\mathbf{q}}\,\ddot{\boldsymbol{s}} \quad (6)$$

Here, $\boldsymbol{\eta} = [\phi\ \theta\ \psi]^T$ is the angular position of the quadrotor, $\boldsymbol{C}$ represents the Coriolis matrix containing the gyroscopic and centripetal terms (Luukkonen, 2011) and $\boldsymbol{\tau}_q$ is the torque vector.

4. Model of a vertical take-off and landing unmanned aircraft vehicle with air-launched missiles

In a conventional quadrotor system, there are four control variables corresponding to four basic movements such as altitude, roll, pitch and yaw movements. These four basic motions of the quadrotor are controlled by changing the propeller speeds accordingly. Also, by rotating the two propeller pairs on the arms with same speed but in opposite direction, the rotational torque can be cancelled. As the propellers are only used for vertical take-off in the unmanned aerial vehicle discussed here, roll, pitch and yaw dynamics will be controlled by propellers only in the vertical motion. In the forward motion of the aircraft, these dynamics are thought to be controlled as in a normal aircraft. By considering the aerial vehicle in this way, its dynamics while performing VTOL movements will be similar to the quadrotor dynamics described in the previous section.

In this study, considering the potential military applications of the VTOL UAV, the total mass of the system will vary throughout the mission. The scenario where air-launched missiles, positioned on the right and left wings of the aircraft, are released at different times will lead to changes in the total mass and inertia tensor of the system during the flight process. The translational dynamics of the VTOL UAV system can be considered as follows:

$$m_t(\ddot{\xi} + g\boldsymbol{i}_z) = \boldsymbol{T} \quad (7)$$

Here, m_t is the total mass of the air vehicle (m_{UAV}) with left (m_{lm}) and right (m_{rm}) air-launched missiles:

$$m_t = m_{UAV} + m_{lm} + m_{rm} \tag{8}$$

The dynamic equation for VTOL UAV for rotational dynamics can be considered as similar to the quadrotor dynamics given in equation (6):

$$\boldsymbol{J}_\mathbf{v}\,\ddot{\boldsymbol{\eta}} + \boldsymbol{C}_\mathbf{v}(\boldsymbol{\eta},\dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} = \boldsymbol{\tau}_\mathbf{v} \tag{9}$$

Here, $\boldsymbol{J}_v$ represents the Jacobian matrix and, $\boldsymbol{C}_v$ represents the Coriolis matrix containing the gyroscopic and centripetal terms similarly to quadrotor dynamics. The fundamental difference between VTOL UAV and quadrotor arises from the acquisition of these matrices. Since the quadrotor is symmetric in both the $-xz$ and $-yz$ planes, its inertia tensor only contains diagonal terms. However, the VTOL UAV system is symmetric only in the $-xz$ plane. This symmetry is disrupted by the release of air-launched missiles on the right and left wings at different times. Therefore, it is necessary to derive the dynamic model of the VTOL UAV system for the general case where the symmetry of the system is disrupted.

Figure 2 provides a top view of the VTOL UAV system under consideration. As seen here, before takeoff, there are air-launched missiles m_{lm} and m_{rm} on the left and right sides. In this study, a VTOL UAV system has been designed to operate in quadrotor mode or perform trajectory tracking in quadrotor mode. In this scenario, the effects of releasing air-launched missiles towards the target at different times on the system dynamics will be observed. Considering the release of these air-launched missiles at different times during flight, three different mass and inertia tensor situations will arise. The total mass and inertia tensor ($\boldsymbol{I}_{VTOL}$) changes corresponding to the three different situations occurring at different times (t_1 and t_2) during the flight (t_f) are provided below:

$$0 < t < t_1 \rightarrow \begin{cases} \text{Total} \quad \text{Mass} = m_{UAV} + m_{lm} + m_{rm} \\ I_{VTOL} = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{zx} & 0 & I_{zz} \end{bmatrix} \end{cases}$$

$$t_1 < t < t_2 \rightarrow \begin{cases} \text{Total} \quad \text{Mass} = m_{UAV} + m_{lm} \\ I_{VTOL} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \end{cases}$$

$$t_2 < t < t_f \rightarrow \begin{cases} \text{Total} \quad \text{Mass} = m_{UAV} \\ I_{VTOL} = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{zx} & 0 & I_{zz} \end{bmatrix} \end{cases} \tag{10}$$

4.1 Euler–Lagrange equations of a vertical take-off and landing unmanned aircraft vehicle

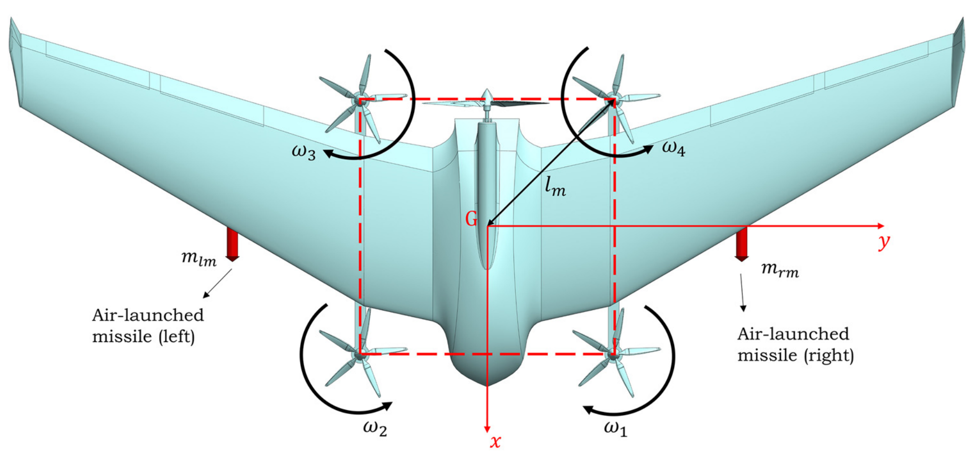
The derivation of the dynamic motion equations for the proposed UAV will be presented in this section using the Euler–Lagrange method. To formulate the Lagrange equation, translational kinetic energy and the potential energy of the system can be expressed as follows:

$$K_{trans} = \frac{1}{2} m_{UAV} \dot{\xi}^\mathbf{T} \dot{\xi}$$
$$U = m_{UAV} g z \tag{11}$$

As the linear and angular components of the UAV are independent, translational and altitude dynamics can be obtained separately. The kinetic and potential energy equations are applied to the given Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial K_{trans}}{\partial \dot{\xi}}\right) - \frac{\partial K_{trans}}{\partial \xi} + \frac{\partial U}{\partial \xi} = T \tag{12}$$

Figure 2 Top view of the VTOL UAV



Source: Figure by authors

After performing the necessary operations in the Lagrange equation, the translational motion equation of the UAV system can be obtained as given in equation (7). To formulate the rotational dynamics of the system, the rotational kinetic energy can be expressed using the Jacobian matrix, as shown below:

$$K_{rot} = \frac{1}{2}\dot{\boldsymbol{\eta}}^{\boldsymbol{T}} \mathcal{J}_v \dot{\boldsymbol{\eta}} \tag{13}$$

The Jacobian matrix can be defined by using the transformation matrix from the inertial frame to the body frame as follows:

$$\boldsymbol{J}_v = \boldsymbol{W}_{\boldsymbol{\eta}}^{\boldsymbol{T}} \boldsymbol{I}_{VTOL} \boldsymbol{W}_{\boldsymbol{\eta}} \tag{14}$$

Here, the Jacobian matrix has been calculated based on the inertia tensor obtained for the case where the symmetric structure of the system is disrupted:

$$\boldsymbol{J}_v = \begin{bmatrix} \mathcal{J}_{11} & \mathcal{J}_{12} & \mathcal{J}_{13} \\ \mathcal{J}_{21} & \mathcal{J}_{22} & \mathcal{J}_{23} \\ \mathcal{J}_{31} & \mathcal{J}_{32} & \mathcal{J}_{33} \end{bmatrix} \tag{15}$$

Below, all elements of the Jacobian matrix are provided in their most general symbolic form as calculated:

$$\begin{aligned}
\mathcal{J}_{11} &= I_{xx} \\
\mathcal{J}_{12} &= \left(\frac{I_{xy}+I_{yx}}{2}\right)C_\phi - \left(\frac{I_{xz}+I_{zx}}{2}\right)S_\phi \\
\mathcal{J}_{13} &= -I_{xx}S_\theta + \left(\frac{I_{xz}+I_{zx}}{2}\right)C_\phi C_\theta + \left(\frac{I_{xy}+I_{yx}}{2}\right)S_\phi C_\theta \\
\mathcal{J}_{21} &= \left(\frac{I_{xy}+I_{yx}}{2}\right)C_\phi - \left(\frac{I_{xz}+I_{zx}}{2}\right)S_\phi \\
\mathcal{J}_{22} &= I_{zz} + (I_{yy}-I_{zz})C_\phi^2 - \left(\frac{I_{yz}+I_{zy}}{2}\right)S_{2\phi} \\
\mathcal{J}_{23} &= -\left(\frac{I_{yz}+I_{zy}}{2}\right)C_\theta + (I_{yz}+I_{zy})C_\phi^2 C_\theta - \left(\frac{I_{xy}+I_{yx}}{2}\right)C_\phi S_\theta \\
&\quad + \left(\frac{I_{xz}+I_{zx}}{2}\right)S_\phi S_\theta + (I_{yy}-I_{zz})C_\phi C_\theta S_\phi \\
\mathcal{J}_{31} &= -I_{xx}S_\theta + \left(\frac{I_{xz}+I_{zx}}{2}\right)C_\phi C_\theta + \left(\frac{I_{xy}+I_{yx}}{2}\right)C_\theta S_\phi \\
\mathcal{J}_{32} &= -\left(\frac{I_{yz}+I_{zy}}{2}\right)C_\theta + (I_{yz}+I_{zy})C_\phi^2 C_\theta \\
&\quad - \left(\frac{I_{xy}+I_{yx}}{2}\right)C_\phi S_\theta + \left(\frac{I_{xz}+I_{zx}}{2}\right)S_\phi S_\theta + (I_{yy} \\
&\quad - I_{zz})C_\phi C_\theta S_\phi \\
\mathcal{J}_{33} &= I_{xx} + (I_{yy}-I_{xx})C_\theta^2 + (I_{zz}-I_{yy})C_\phi^2 C_\theta^2 - (I_{xz}+I_{zx})C_\phi C_\theta S_\theta \\
&\quad - (I_{xy}+I_{yx})C_\theta S_\phi S_\theta + (I_{yz}+I_{zy})C_\phi C_\theta^2 S_\phi
\end{aligned} \tag{16}$$

The rotational kinetic energy equations are applied to the given Lagrange equation to obtain the rotational dynamics of the VTOL UAV:

$$\frac{d}{dt}\left(\frac{\partial K_{rot}}{\partial \dot{\boldsymbol{\eta}}}\right) - \frac{\partial K_{rot}}{\partial \boldsymbol{\eta}} = \boldsymbol{\tau}_v \tag{17}$$

After performing the necessary operations in the Lagrange equation, the rotational motion equation of the UAV system can be obtained as given in equation (9). In this equation $\boldsymbol{C}_{\mathbf{v}}(\boldsymbol{\eta},\dot{\boldsymbol{\eta}})$ is a Coriolis matrix, containing the gyroscopic and centripetal terms:

$$\boldsymbol{C}_{\mathbf{v}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{18}$$

The elements of the Coriolis matrix for the case where the UAV is not symmetric are listed below:

$$\begin{aligned}
C_{11} &= 0 \\
C_{12} &= \dot\theta\left(\frac{I_{yx}+I_{zy}}{2}\right)C_{2\phi} + \dot\theta\left(\frac{I_{yy}+I_{zz}}{2}\right)S_{2\phi} - (I_{xx}+I_{zz})\dot\psi C_\theta - (I_{xz}+I_{zx})\dot\psi C_\phi S_\theta - (I_{xy}+I_{yx})\dot\psi S_\phi S_\theta + (2I_{zz}C_\phi^2 - I_{yy}C_{2\phi})\dot\psi C_\theta + 2(I_{yz}+I_{zy})\dot\psi C_\phi C_\theta S_\phi \\
C_{13} &= -\dot\psi(I_{yz}-I_{zy})C_\phi^2 C_\theta^2 + \dot\psi\left(\frac{I_{xy}+I_{yx}}{2}\right)C_\phi C_\theta S_\theta - \dot\psi\left(\frac{I_{xz}+I_{zx}}{2}\right)C_\theta S_\phi S_\theta + \dot\psi\left(\frac{I_{yz}+I_{zy}}{2}\right)C_\theta^2 + \dot\psi(I_{zz}-I_{yy})C_\phi C_\theta^2 S_\phi \\
C_{21} &= -\left(\frac{I_{xz}+I_{zx}}{2}\right)C_\phi\dot\phi - \left(\frac{I_{xy}+I_{yx}}{2}\right)S_\phi\dot\phi + (I_{xx}-I_{yy}+I_{zz})\dot\psi C_\theta + (I_{xz}+I_{zx})\dot\psi C_\phi S_\theta + (I_{xy}+I_{yx})\dot\psi S_\phi S_\theta + 2(I_{yy}-I_{zz})\dot\psi C_\phi^2 C_\theta - 2(I_{yz}-I_{zy})\dot\psi C_\phi C_\theta S_\phi \\
C_{22} &= (I_{yz}+I_{zy})\dot\phi - 2(I_{yz}+I_{zy})\dot\phi C_\phi^2 + (I_{zz}-I_{yy})\dot\phi S_{2\phi} \\
C_{23} &= -\left(\frac{I_{xz}+I_{zx}}{2}\right)C_\phi\dot\psi - \left(\frac{I_{xy}+I_{yx}}{2}\right)S_\phi\dot\psi + \left(\frac{I_{yy}-I_{xx}}{2}\right)S_{2\theta}\dot\psi + (I_{xz}+I_{zx})C_\phi C_\theta^2\dot\psi + (I_{xy}+I_{yx})C_\theta^2 S_\phi\dot\psi + (I_{zz}-I_{yy})C_\phi^2 C_\theta S_\theta\dot\psi + (I_{yz}+I_{zy})C_\phi C_\theta S_\phi S_\theta\dot\psi \\
C_{31} &= \left(\frac{I_{xy}+I_{yx}}{2}\right)C_\phi C_\theta\dot\phi - \left(\frac{I_{xz}+I_{zx}}{2}\right)C_\theta S_\phi\dot\phi + (I_{zz}-I_{xx}-I_{yy})\dot\theta C_\theta + 2(I_{yy}-I_{zz})\dot\theta C_\phi^2 C_\theta - 2(I_{yz}+I_{zy})\dot\theta C_\phi C_\theta S_\phi \\
C_{32} &= \left(\frac{I_{yz}+I_{zy}}{2}\right)S_\theta\dot\theta - \left(\frac{I_{xy}+I_{yx}}{2}\right)C_\phi C_\theta\dot\theta + \left(\frac{I_{xz}+I_{zx}}{2}\right)C_\theta S_\phi\dot\theta - (I_{yz}+I_{zy})C_\phi^2 S_\theta\dot\theta + (I_{zz}-I_{yy})C_\phi S_\phi S_\theta\dot\theta \\
C_{33} &= (I_{xz}+I_{zx})\dot\theta C_\phi + (I_{xy}+I_{yx})\dot\theta S_\phi - (I_{yz}+I_{zy})\dot\phi C_\theta^2 + (I_{xx}-I_{yy})\dot\theta S_{2\theta} - 2(I_{xz}+I_{zx})\dot\theta C_\phi C_\theta^2 - 2(I_{xy}+I_{yx})\dot\theta S_\phi C_\theta^2 + 2(I_{yz}+I_{zy})\dot\phi C_\theta^2 C_\phi^2 - (I_{xy}+I_{yx})\dot\phi C_\phi C_\theta S_\theta \\
&\quad + (I_{xz}+I_{zx})\dot\phi C_\theta S_\phi S_\theta + 2(I_{yy}-I_{zz})\dot\phi C_\phi C_\theta^2 S_\phi + 2(I_{yy}-I_{zz})\dot\theta C_\phi^2 C_\theta S_\theta - 2(I_{yz}-I_{zy})\dot\theta C_\phi C_\theta S_\phi S_\theta
\end{aligned} \tag{19}$$

In the translational and rotational motion equations given in equation (7) and (9) for the VTOL UAV, $\boldsymbol{T} = \boldsymbol{R}\ [0\ 0\ T]^T$ represents the total thrust vector, while $\tau_q = [\tau_\phi\ \tau_\theta\ \tau_\psi]^T$ denotes the total torque expression. Thrust and torque expressions, expressed in terms of the angular velocities of the rotors, can be written as follows here:

$$\begin{aligned} T &= l_m k\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right) \\ \tau_\phi &= sin\left(\frac{\pi}{4}\right) l_m k\left(\omega_1^2 - \omega_2^2 - \omega_3^2 + \omega_4^2\right) \\ \tau_\theta &= sin\left(\frac{\pi}{4}\right) l_m k\left(-\omega_1^2 - \omega_2^2 + \omega_3^2 + \omega_4^2\right) \\ \tau_\psi &= b\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right) \end{aligned} \tag{20}$$

In an aircraft with four rotors, there are two different configurations: plus (+) and cross (×). The quadrotor in Figure 1(b) has a plus configuration. In this configuration, the four rotors are arranged in the shape of a plus sign. This type of configuration is often used to improve balance and stability by using a four rotor with plus arrangement. The VTOL UAV depicted in Figure 2, which is the subject of this study, has a cross configuration. In this configuration, the rotors are arranged in a cross shape. The four rotors are positioned diagonally across the center of the aircraft. This arrangement is typically chosen to enhance aerodynamic performance and maneuverability. The expressions for thrust and torque of the VTOL UAV system, as provided in equation (18), have been obtained based on the cross configuration.

5. Trajectory control simulation of a vertical take-off and landing unmanned aircraft vehicle with air-launched missiles

The VTOL UAV system has been modeled in this study by drawing an analogy to the quadrotor structure. The aircraft, with its VTOL capability, can follow a desired trajectory during takeoff, landing, or in-flight operations. The desired trajectory created for the simulation study is illustrated in Figure 3. The trajectory of VTOL UAV, rising from the ground to an altitude of 50 meters within 240 s, is assumed to be followed.

The physical parameters of the VTOL UAV system used in the simulation study are presented in Table 1. In this context, the aircraft, with a total payload capacity of 4 kg, is considered to carry two lightweight 2 kg air-launched missiles on each of its right and left wings. For the first flight scenario of the VTOL UAV system, it is envisaged that, within the desired trajectory lasting 240 s, air-launched missiles located on the right and left wings will be simultaneously released at the 160th second. In addition, another flight scenario has been devised for a VTOL UAV system, where a right air-launched missile is released at the 160th second, followed by the release of a left air-launched missile at the 200th s, within the 240-s desired trajectory. Furthermore, through these two scenarios, the minimum time required for the aircraft to transition to a stable position when releasing two air-launched missiles at different times has been determined.

The VTOL feature is implemented here with a quadrotor structure, and control design has been carried out. A classical PID controller is designed for trajectory control. Controller designs for both translational and rotational dynamics have been implemented separately, as they can be obtained independently of each other. The control structure, which has been simulated, is illustrated in Figure 4. VTOL UAV system only has four actuators to control its desired trajectory, but overall system has six degrees of freedom therefore it is considered as an underactuated system. The total thrust vector ($\boldsymbol{T}$) and total torque vector (τ_v), calculated with the PID controller, constitute the inputs to the system. As depicted in Figure 4, the system takes the desired trajectory as input; however, the desired roll and pitch movements for rotational dynamics are calculated through the translational controller.
The selected control gains for the PID controller gains for translational control of the system are as follows:

$$\begin{aligned} K_{p,t} &= 25 \\ K_{i,t} &= 8 \\ K_{d,t} &= 110 \end{aligned} \tag{21}$$

Similarly, PID controller gains for rotational control are as follows:

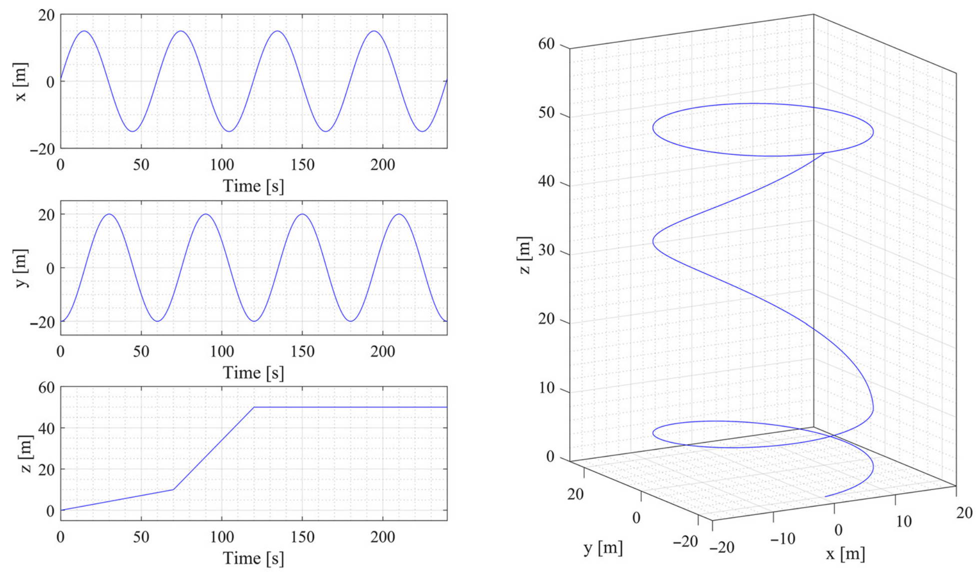
$$\begin{aligned} K_{p,r} &= 2 \\ K_{i,r} &= 0.1 \\ K_{d,r} &= 5 \end{aligned} \tag{22}$$

In Figure 5, it can be observed that the VTOL UAV system successfully tracks the desired trajectory through PID control for the first scenario of the flight. At the halfway point of the total flight time, i.e. at the 120th s, the UAV reaches its intended circular trajectory in the air. While following this circular path, it releases both left and right air-launched missiles at the 160th simultaneously. While the simultaneous release of right and left air-launched missiles does not disrupt the symmetric structure of the system, changes in system parameters, particularly mass variations, can disturb the targeted altitude during the flight. To observe its effect more clearly on the system, Figure 6 presents the altitude graph of the UAV. The release of missiles by the UAV has momentarily created a disruptive effect. As evident from the subplot provided in Figure 6, the simultaneous release of both air-launched missiles induces an approximate trajectory deviation of 0.83 meters on the aircraft.

In addition to the simultaneous release of air-launched missiles during the total flight duration, releasing them at different times may also be part of the aircraft's mission objectives. Therefore, a second flight scenario has been investigated, involving the release of the missile located on the right wing at the 160th s and the one on the left wing at the 200th s. Figure 7 illustrates the altitude variation of the aircraft for the second scenario.

An aircraft typically uses its capability not to release the carried air-launched missiles simultaneously, but rather to strike different targets at different times. Therefore, it is crucial for the aircraft to maintain a stable position after releasing the first air-launched missile to successfully target the second one at a different objective. In this scenario, a simulation has been conducted to demonstrate the number of seconds it takes for the aircraft to release the second air-launched missile after the initial release of the missile. As seen in Figure 7, releasing a single air-launched missile has a less disruptive effect on the

Figure 3 Desired trajectory of VTOL UAV



Source: Figure by authors

Table 1 Parameters of the VTOL UAV system

Parameter	Definition	Value and [unit]
m	Mass of the UAV	$0 < t < t_1 \rightarrow 36.47$ [kg] $t_1 < t < t_2 \rightarrow 34.47$ [kg] $t_2 < t < t_f \rightarrow 32.47$ [kg]
m_{lm}	Mass of the left Air-launched missile	2 [kg]
m_{rm}	Mass of the right Air-launched missile	2 [kg]
I_{VTOL}	Inertia tensor of the UAV	$0 < t < t_1 \rightarrow \begin{bmatrix} 10.4292 & 0 & 0.0570 \\ 0 & 1.4340 & 0 \\ 0.0570 & 0 & 11.8074 \end{bmatrix} [kgm^2]$ $t_1 < t < t_2 \rightarrow \begin{bmatrix} 9.3780 & 0.1684 & 0.0429 \\ 0.1684 & 1.3254 & 0.0872 \\ 0.0429 & 0.0872 & 10.6626 \end{bmatrix} [kgm^2]$ $t_2 < t < t_f \rightarrow \begin{bmatrix} 8.3268 & 0 & 0.0288 \\ 0 & 1.2168 & 0 \\ 0.0288 & 0 & 9.5178 \end{bmatrix} [kgm^2]$
l_m	Distance between rotors and UAV center of mass	0.5 [m]
k	Lift constant	1.2 [–]
b	Drag constant	0.083 [–]
g	Acceleration of gravity	9.81 [m/s^2]

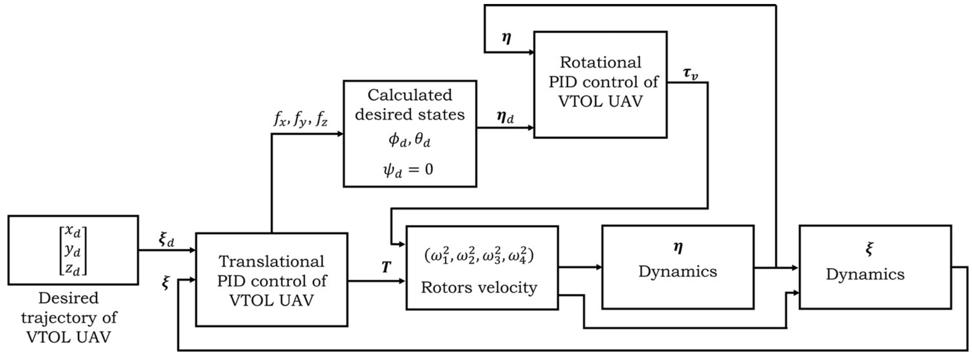
Source: Table by authors

aircraft compared to releasing both of them simultaneously. The release of the single air-launched missiles induces an approximate trajectory deviation of 0.42 meters on the aircraft. For the next air-launched missile to reach its target successfully, it is crucial for the VTOL UAV to attain a stable position. Based on the simulation results, it has been observed that this duration should be at least 25 s. After 25 s, the aircraft is able to resume tracking the desired trajectory. This results in a more stable shot for the air-launched missiles released from the aircraft for the second time.

6. Conclusions

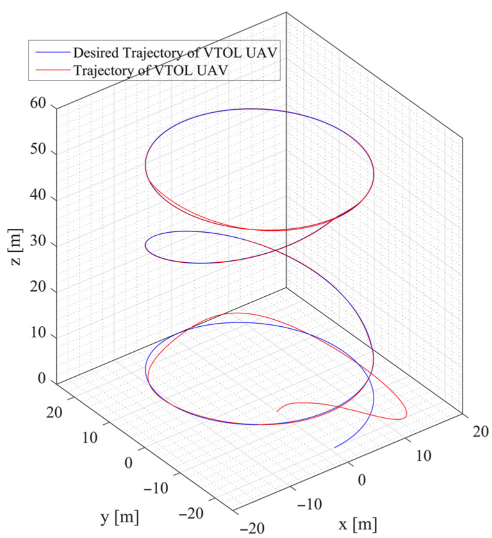
In this study, the mathematical model of an aircraft with a critical VTOL feature, particularly significant in military applications, has been obtained using the Lagrange approach. This model provides a fundamental framework for the precise control and determination of the trajectory for VTOL UAV systems. Therefore, a simulation study has been conducted by applying trajectory control to the dynamic model for VTOL UAV systems. This application is a crucial step to ensure the

Figure 4 Schematic control diagram of the VTOL UAV



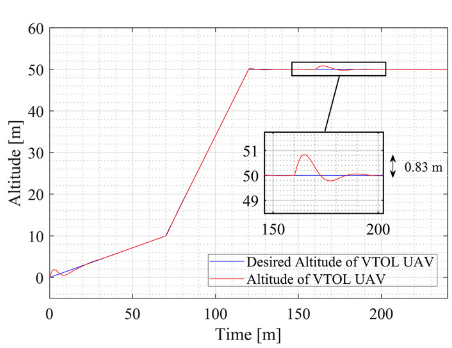
Source: Figure by authors

Figure 5 Desired and actual trajectory of VTOL UAV for releasing the air-launched missiles simultaneously



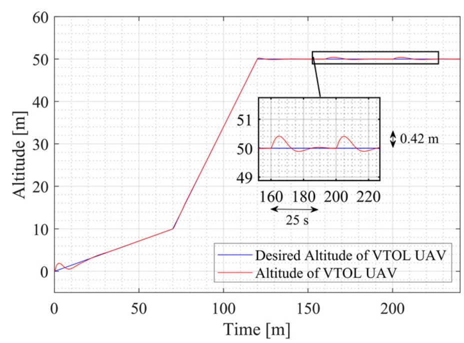
Source: Figure by authors

Figure 6 Altitude of VTOL UAV for releasing the air-launched missiles simultaneously



Source: Figure by authors

Figure 7 Altitude of VTOL UAV for releasing the air-launched missiles different time



Source: Figure by authors

stability of VTOL UAV systems during flight and to guide them toward their targets. Unlike other studies in the literature, it has been demonstrated that the parameters of the considered VTOL UAV system change throughout the flight duration. This innovation aims to create a model that better reflects real-world conditions. Additionally, the time interval between air-launched missiles released at different times has been examined in terms of the aircraft's ability to retrace its desired trajectory.

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