



Magnetic field mapping simulations of sub-mm period undulators for the next generation X-ray free electron lasers

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ABSTRACT

There is a growing interest on the compact XFELs to be used in applications such as medical science (surgery, fat removal), material science and military. In order to build a compact XFEL, constructing an undulator with sub-millimeter period (λ_u) is mandatory. Many problems arise due to miniaturization of undulators; such as keeping the magnetic field high while still having a considerable gap to minimize beam scraping. In this work, RADIA program has been employed for the modeling of sub millimeter period undulators ($20 \mu\text{m} < \lambda_u < 400 \mu\text{m}$) with three different magnet configuration; namely Up-Down ($\uparrow\downarrow\uparrow\dots$), Halbach (Wiggler) ($\uparrow\rightarrow\downarrow\leftarrow\uparrow\dots$) and Hybrid ($\rightarrow\uparrow\leftarrow\downarrow\rightarrow\dots$). Effects of sub-millimeter undulator period and gap on the magnetic field pattern of the undulator have been discussed.

1. Introduction

Understanding the fundamental processes in atomic or molecular level, will pave the way to better technological advancements. John Madey invented a powerful tool, Free Electron Laser (FEL), in 1971. [1] FEL shed light to a world, which is at the same time ultra-small (below 100 nm) and ultra-fast (down to fs) such as the hydrogen transfer time of the molecules (~ 1 ns), the spin precession time (~ 10 ps) and computing time (~ 1 ns). [2–8]

In a typical FEL, beam of electron is accelerated to almost the speed of light in an electron accelerator. E-beam then passes through an undulator [9]; an insertion device consists of two parallel arrays of magnets with alternating polarity ($\uparrow\downarrow\uparrow\dots$), facing each other with an air gap in between. Due to the alternating pattern of the magnetic field, electrons follow a sinusoidal path instead of going straight. These oscillations force electrons to radiate energy. FEL generates tunable, coherent, high power radiation, currently spanning wavelengths from millimeter to X-ray (XFEL). [10]

One of the important parameters of the XFEL is the undulator period (λ_u , distance between the midpoints of two consecutive magnets with same polarity), which has a direct effect on the X-ray radiation. Current state of the art of XFEL's; LCLS/SLAC uses undulator period of 3 cm and generates an output radiation wavelength down to 1.5 Å using e-beam with energies ~ 14 GeV. [11] However, this kind of high-energy e-beam could only be generated on large accelerator facilities. Therefore, applications of FELs are limited to very special cases such as the observation of photosynthesis, spin recession under the magnetic field, and the chemical reactions. Recently, critical eye and brain tumor surgeries were also performed in FEL with minimal damage to adjacent tissues and without any secondary damage. In addition, many disciplines such as material science want the FEL to be used in laboratory

environment such as TEM and XRD. Besides, the fact that FELs are able to emit high-energy electromagnetic radiation and therefore a potential weapon attracts the attention of the defense industry. [12–16] Reducing the undulator period from 3 cm to 300 μm would reduce the required e-beam energy to 1.4 GeV. Consequently, more compact e-beam sources could be used and make XFELs readily available for special applications. In a different perspective, short-period undulator would generate higher energy radiation, when coupled with high-energy electron accelerators, such as gamma rays, which would pave the way to new discoveries in science.

Here we draw a complete picture of the influence of sub millimeter undulator period, gap and magnetic configuration on the magnetic field wave of the undulator by using modeling.

2. Simulation parameters

Undulators with periods ranging from 20 μm to 400 μm and gap (the gap between the two parallel magnet arrangement) ranging from 10 μm to 100 μm have been simulated by the Mathematica extension RADIA [17]. Three different magnetic configurations are considered; Up_Down (the standard consecutive north/south pole array ($\uparrow\downarrow\uparrow\dots$)), Halbach (Wiggler undulator, compared to the standard sequence, provides a higher magnetic field and the magnetic field oscillates perpendicular to the e-beam direction. Thus, a high-fluence and energetic, but broad-spectrum radiation beam is obtained.) [18] ($\uparrow\rightarrow\downarrow\leftarrow\uparrow\dots$) and Hybrid (magnetically hard phase combined with high permeability soft phase ($\rightarrow\uparrow\leftarrow\downarrow\rightarrow\uparrow\leftarrow\dots$)). (Fig. 1)

Micro magnets of Nd-Fe-B with remanent magnetization (Mr) of 1.3 T have been used in the simulated undulators (Fig. 2). Hybrid magnets

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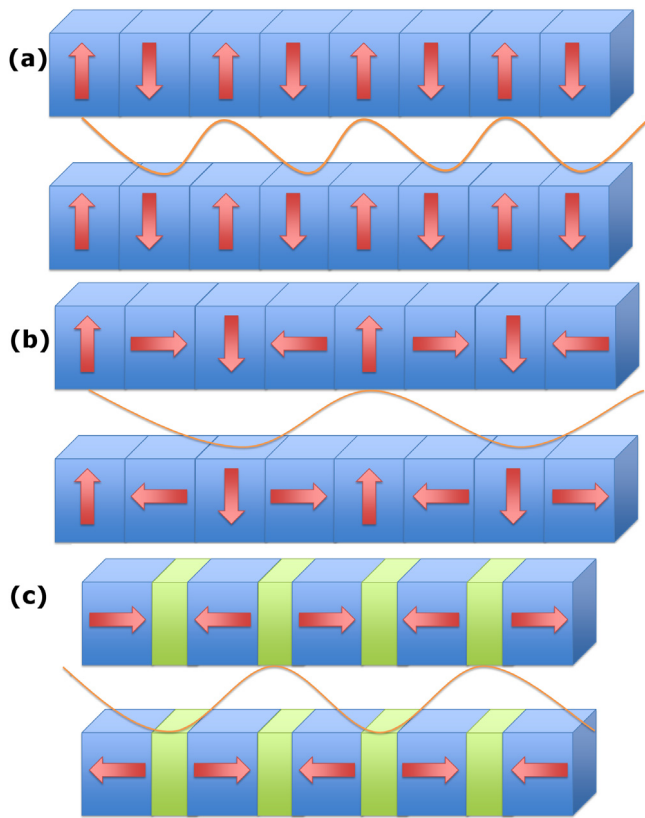


Fig. 1. (color online) Magnetic configurations of undulators used in simulations: (a) Up-Down, (b) Halbach and (c) Hybrid.

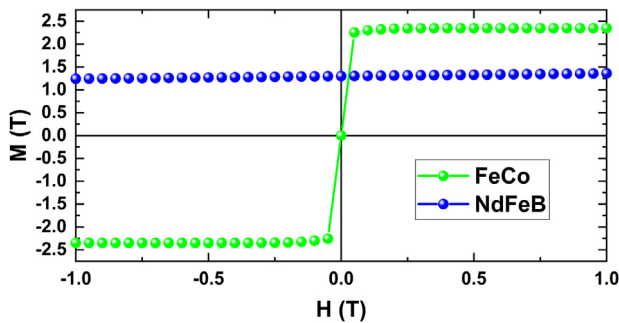


Fig. 2. (color online) Hysteresis loops of Nd-Fe-B micro-magnets and FeCo micro-poles used in simulations.

are composed of Nd-Fe-B micro magnets and soft magnetic FeCo (Fe: 74.2, Co: 25%, Cr: 0.3%, Mn: 0.5%) poles to channel magnetic field through high permeability path. Thickness (in the direction of e-beam) of the micro magnets and poles is between 10 μm to 100 μm and 10 μm , respectively. Width and height, 100 μm \times 100 μm , of micro magnets are kept constant all throughout the simulation. Effects of undulator parameters in the sub millimeter regime on sinusoidal magnetic field inside the undulator are studied.

2.1. Results and discussion

Fundamental parameter for characterizing an undulator, K , is defined as:

$$K = 0.93 B_0 [T] \lambda_u [\text{cm}] \quad (1)$$

where B_0 is the peak value of the sinusoidal magnetic field. Wavelength of radiation obtained from FEL, λ_{rad} is;

$$\lambda_{\text{rad}} \cong \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) \quad (2)$$

where γ is the Lorentz factor of the relativistic electrons. [10] Depending on the application, FEL would produce radiation with a wavelength ranging from millimeter to the X-ray's and even Gamma rays. 20 GeV of energy ($\lambda_u = 3$ cm and $K = 2$) is required to obtain radiation with a wavelength of 1 \AA (solid X-ray). The energies of this magnitude are possible only with very large size, high cost and complex electron accelerators and elaborate undulators. It is essential to reduce the FEL's dimensions. In the case of the small undulator period, $K \ll 1$, Eq. (2) is reduced to $\lambda_{\text{rad}} = \lambda_u / 2\gamma^2$. In other words, the energy required to generate radiation at a wavelength of 1 \AA is reduced by 10 times to 2 GeV, by reducing the undulator period by 100 times from 3 cm to 300 μm . Thus, compact FELs can be built with much smaller electron accelerators. Such compactness is also achieved via inverse-Compton scattering by using the laser as an undulator, however, the system is complicated, costly and hard to maintain, not to mention the energy of the output of the radiation is small. [19] In this work, authors focus on the conventional undulators in smaller scale.

The reduction of the undulator period depends on the reduction of the magnet size by maintaining the magnetic properties. Three kinds of methods are used to provide the magnetic field in the undulators; micro-electromagnets, [20,21] superconductors [22] and permanent magnets [23–26]. Electromagnets; complicated designs are needed to supply electrical power and cooling system is necessary. The superconductors are also not suitable for an undulator design, since they also require a lot of energy for the cooling of the system. Even though the magnetic field cannot be adjusted on permanent magnets, it is ideal for the development of a sub millimeter-period undulator due to their simple design, requiring no power and cooling system. Considering the advances in latest nano / micro magnet synthesis technologies such as lithography, this work is focused on permanent magnets.

Generating periodic magnetic fields in sinusoidal shape is one of the main requirements of an undulator. Thus, sinusoidal pattern has to be maintained while miniaturizing the undulator, which will ensure high quality output radiation. [18] Sinusoidal magnetic field inside the undulator is;

$$B = B_0 \sin\left(\frac{2\pi y}{\lambda_u}\right) \quad (3)$$

Sub-millimeter undulator has been simulated and snapshots from the magnetic field wave inside the undulator for Up-Down, Hybrid and Halbach configuration has been taken at different magnet thicknesses (t) and undulator gaps. (Figs. 3–5)

2.2. Up-Down configuration

Magnetic field wave snapshots of up-down configuration reveal an evolution from triangle to square waveform (non-sinusoidal waveforms) between 10 to 100 μm magnet thickness at 10 μm undulator gap. (Fig. 3) Sinusoidal wave could be recovered by increasing the gap at the expense of maximum magnetic field. In order to acquire coherent radiation single frequency sine waveform is preferred. [18] Non-sinusoidal waveforms are the interference of multiple sine waves of different frequencies while pure sine waves consist of a single frequency. The frequency and amplitude of each component can be found using Fourier analysis. The fast Fourier transform (FFT) or just Fourier transform decomposes a wave into frequencies. As an example, undulator with magnet thickness of 100 μm is chosen for FFT analysis due to its square waveform. FFT of magnetic field waves taken at 10, 50, 100 and 150 μm gaps is shown in Fig. 6 a. While the FFT of 10 μm undulator gap with square wave gives peaks at ten different frequencies, the number of peaks decreases as the gap increases thus the wave form transforms to sinusoidal form. Thereupon, sinusoidal wave with single frequency is

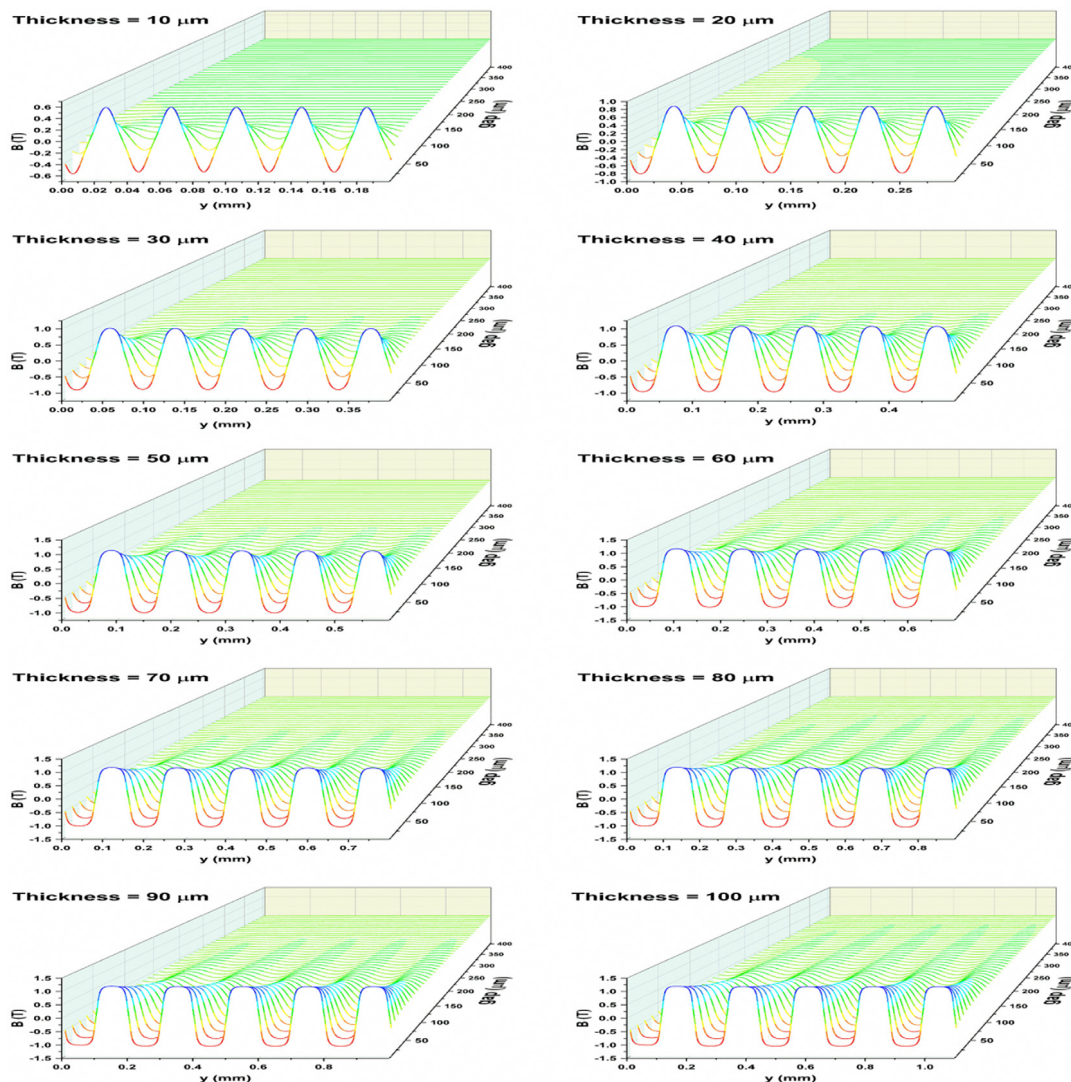


Fig. 3. (color online) Magnetic field snapshots of Up_Down configuration: Snapshots has been taken for micro magnet thickness between 10 to 100 μm and for each thickness undulator gap has been scanned through 10 to 400 μm .

obtained at 150 μm undulator gap. However, maximum magnetic field B_0 decays rapidly down to ~ 0.1 T. This finding is supported by the experimental trials on the sub millimeter period. [25] Consequently, two problems arise while miniaturizing the undulator; while decreasing the magnet thickness thus the undulator period, undulator gap has to be small to preserve high magnetic field values. On the other hand, as stated above, reducing the gap results in the deterioration of the sinusoidal waveform. Recovering the sinusoidal waveform requires a gap enlargement, which in turn results in the decay of the magnetic field.

2.3. Hybrid configuration

In order to counterbalance the magnetic field lost during the gap increase in Up-Down configuration, hybrid configuration has been simulated. In a hybrid undulator, high permeability soft magnet (FeCo alloy) is sandwiched between two permanent magnets magnetized oppositely with respect to each other along the y coordinate ($\rightarrow\leftarrow$). Consequently, higher magnetic field values could be achieved by channeling the magnetic field through a high permeability path. Snapshots of magnetic field wave for all magnet thicknesses and gap combinations reveal an ambiguous behavior. (Fig. 4) Waveform for 10 μm thickness and gap is sinusoidal (with a maximum magnetic field value of little

less than 1 T) thus preferred however increasing the thickness while keeping the gap constant alters the waveform from sinusoidal to spiked. This kind of wave has at least the superposition of two different waves, hence not preferred. Highest magnetic field value up to 2.5 T is achieved in undulators with 100 μm thickness magnets ($\lambda_u = 220$ μm) and 10- μm gap. Hence, the loss due to short λ_u and/or wide gap would be compensated much more efficiently. In each thickness value sinusoidal wave is recovered by increasing the gap. Similar to the study performed in the up_down configuration, an undulator with a magnet thickness of 100 μm is chosen for FFT analysis due to its spiked waveform. (Fig. 6b) FFT of 10 μm undulator gap reveals that the spiked waveform is composed of 10 different waves with diverse frequencies. The amplitudes (maximum magnetic fields) of the components with different frequencies were shown to be higher compared to the up-down configuration, as expected. Although the maximum field achieved on the magnetic field wave is 2.5 T, FFT analysis shows that the maximum field achieved on individual components is little over 1.25 T.

2.4. Halbach configuration

In the context of this paper, effect of sub-millimeter periods on the Halbach configuration is also simulated. (Fig. 5) Sinusoidal waveform with maximum field, B_0 of ~ 1 T has been observed at 10 μm magnet

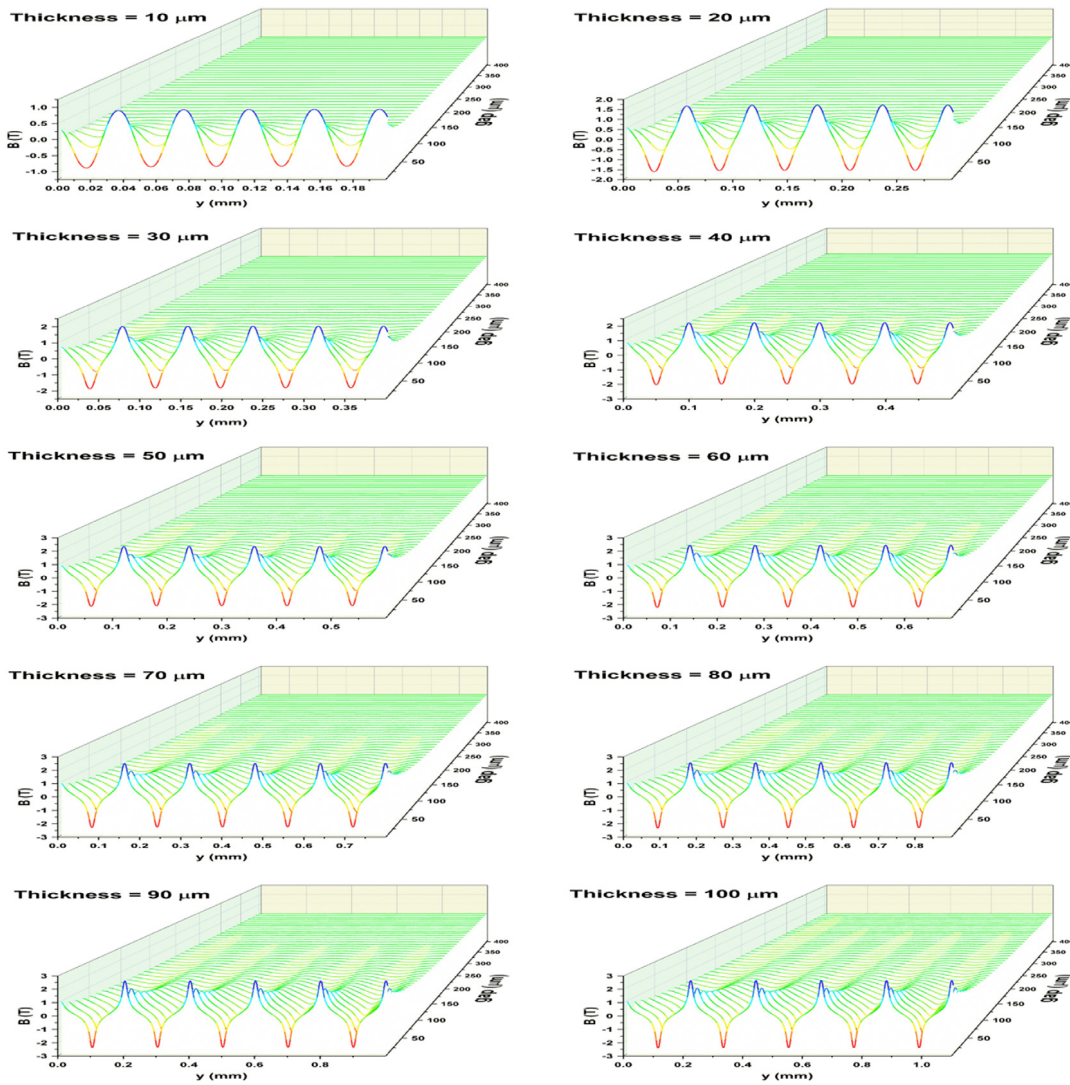


Fig. 4. (color online) Magnetic field snapshots of Hybrid configuration: Snapshots has been taken for micro magnet thickness between 10 to 100 μm and for each thickness undulator gap has been scanned through 10 to 400 μm .

thickness ($\lambda_u = 40 \mu\text{m}$) and 10 μm undulator gap. FFT of the wave gives single frequency, thus suitable for generating coherent radiation. (Fig. 7) Keeping the gap value constant and increasing the thickness result in the deformation of the waveform due to superposition of the waves. For each thickness value Sinusoidal waveform is recovered by increasing the gap at the expense of magnetic field, which is comparable to previous configurations. Analogous to the aforementioned study above; complete FFT analysis was performed on simulated undulators composed of magnets with 100 μm thickness and four different gaps; 10, 50, 100 and 150 μm . (Fig. 6c) FFT analysis of the magnetic field wave, simulated from the undulator with a 10 μm gap, revealed that the wave composes of eight different waves with different frequencies and amplitudes. Single frequency wave was observed only for an undulator with a 150 μm gap with a maximum magnetic field of 0.3 T.

In pursuance of constructing a complete picture of sub-millimeter undulators, maximum peak field, B_0 , versus undulator period λ_u at different gaps (Fig. 8) and B_0 versus gap at different magnet thicknesses (Fig. 9) are plotted for three configurations. In the first case, B_0 plateaus to a maximum value by increasing the undulator period at each gap value. Increasing the gap results in the decay of magnetic field.

Same behavior from different perspective is seen from the latter case (B_0 vs. gap; Fig. 9). Maximum peak field exponentially fades by increasing the gap. While the fastest decay has been observed for the case of hybrid configuration, Halbach configuration decays the slowest.

It is expected that the implementation of the sub-millimeter period undulators would be through a consequent sequence of short segments due to the known limitations of the production. Thus, the impact of the behavior of end fields would be tremendous both between the segments and at the exit, since end fields have a direct impact on the beam exit position and angular divergence relative to the entrance position. Consequently, the beam could be corrected if necessary. The behavior has been studied for all configurations by performing simulations on two consecutive segments; segments were constructed by taking the magnet thickness (30 μm for up-down configuration) and gap 10 μm . The separation distance between the segments was swept between 10 to 100 μm . Magnetic field snapshots, taken at 10 μm and 50 μm separation distance (see the supplemental videos for a complete sweep of separation distance), could be seen in Fig. 10. The distortion of the end field is most pronounced in the hybrid configuration (Fig. 10b). Furthermore, while the distortion at 10 μm segment separation is negligible, it is detrimental to the beam at 50 μm . The distortion of the end field is comparably minor for Halbach configuration, however, there is a similar detrimental distortion behavior in between segments (Fig. 10c). Minimum distortion has been observed for up-down configuration, thus minimal correction to the beam would be needed.

Our calculations on the magnetic field pattern of the sub millimeter period shed light to the scattered data obtained by the recent works on the production of working sub millimeter period undulator [20–27].

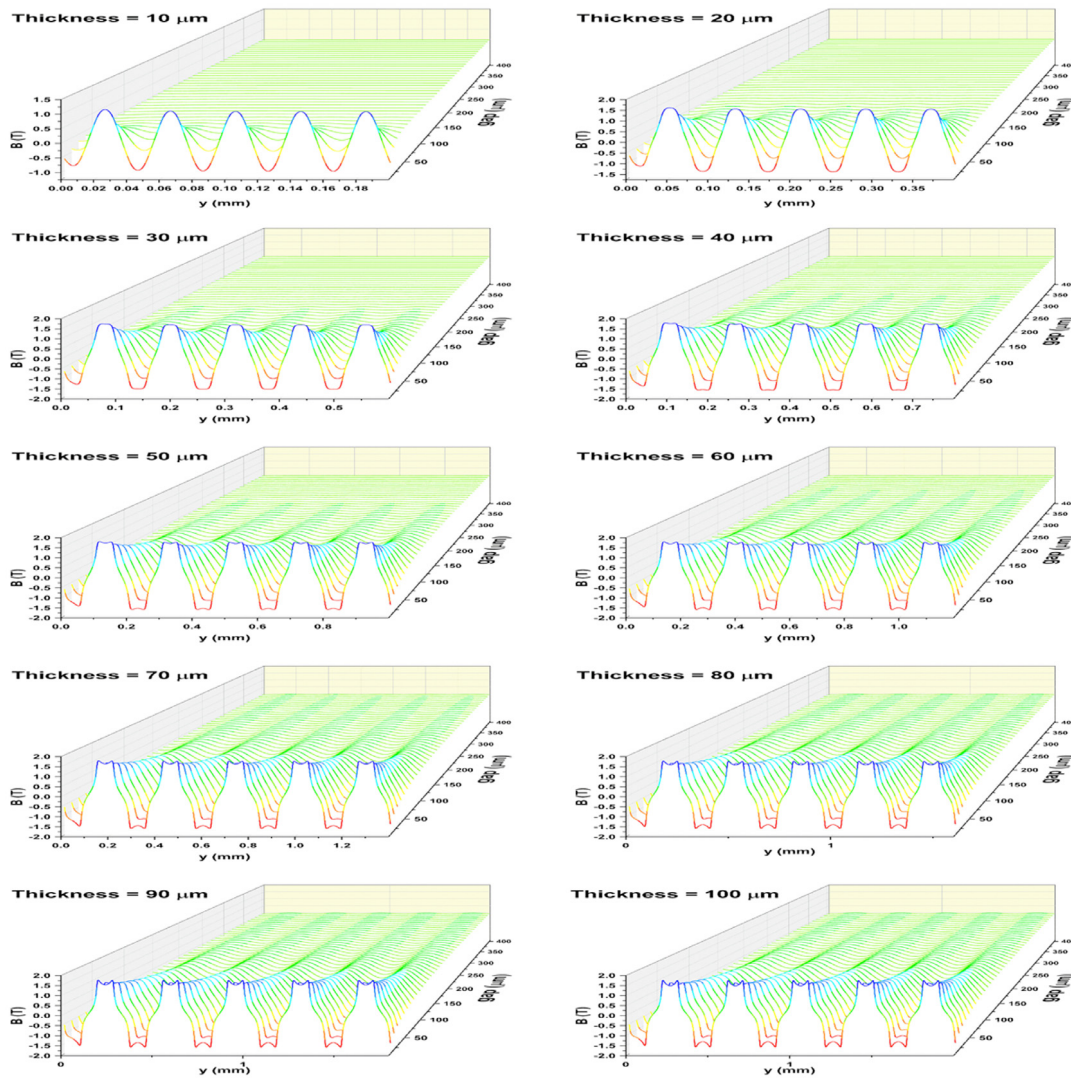


Fig. 5. (color online) Magnetic field snapshots of Halbach configuration: Snapshots has been taken for micro magnet thickness between 10 to 100 μm and for each thickness undulator gap has been scanned through 10 to 400 μm .

3. Conclusions

In conclusion, field behavior of the undulators has been simulated for undulator periods below 500 μm . Three different magnet configurations have been tested; Up-Down, Hybrid and Halbach. Highest magnetic field of 2.5 T has been achieved for Hybrid configuration at 10 μm gap and 100 μm magnet thickness. In Halbach and Hybrid configurations sinusoidal waveform has been observed for the lowest magnet thickness thus the undulator period. For all cases, magnetic field rapidly decays by increasing the gap, but the slowest decay has been observed in Halbach configuration. At each particular gap value B_0 value plateaus to a max field. One final consideration would be the sinusoidal behavior of the waves, which deteriorates at low gap values and slowly heals back to sinusoidal waveform at higher gap values. Additionally, two consecutive undulator segments were also simulated to study the behavior of the end field and the field in between undulator segments; distortion of the field was found to be highest for Hybrid configuration. In the view of the results of this work, we draw a complete picture of the shortcoming of the miniaturization of undulators via modeling. Technological advancements on making smaller and better magnets, opens the door to compact undulators for the next generation X-FELs.

CRediT authorship contribution statement

N.G. Akdogan: Conceptualization, Methodology, Writing - original draft. **O. Polat:** Investigation, Software, Conceptualization, Methodology, Writing - original draft, Writing - review & editing. **O. Akdogan:** Conceptualization, Methodology, Supervision, Funding acquisition, Project administration, Writing - original draft, Writing - review & editing, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.nima.2020.164062>.

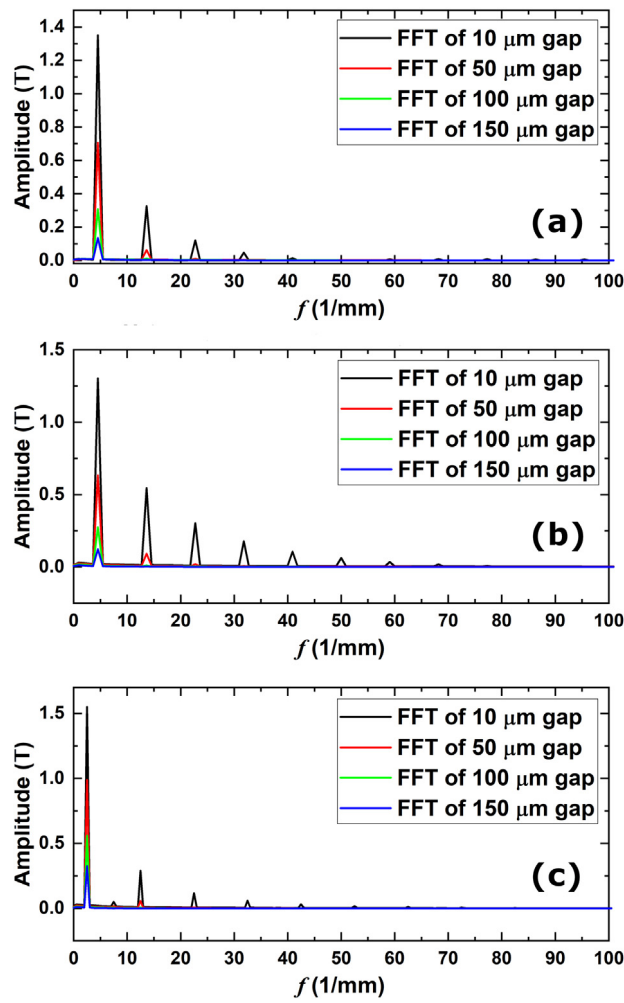


Fig. 6. (color online) Figure shows the FFT of Magnetic field waves at various gap values for 100 μm magnet thickness in (a) up-down,(b) hybrid, and (c) Halbach configurations.

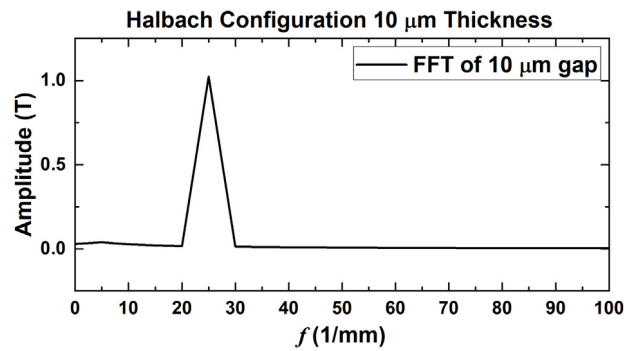


Fig. 7. Figure shows the FFT of Magnetic field wave at 100 μm magnet thickness and 100 μm gap in Halbach configuration undulator.

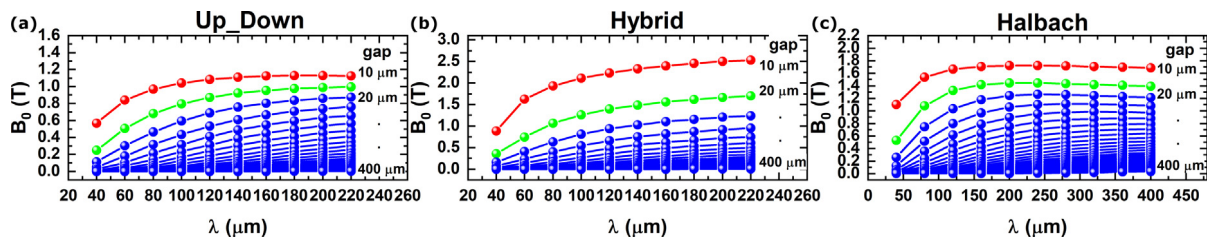


Fig. 8. (color online) Reaction of B_0 to change in undulator period and gap.

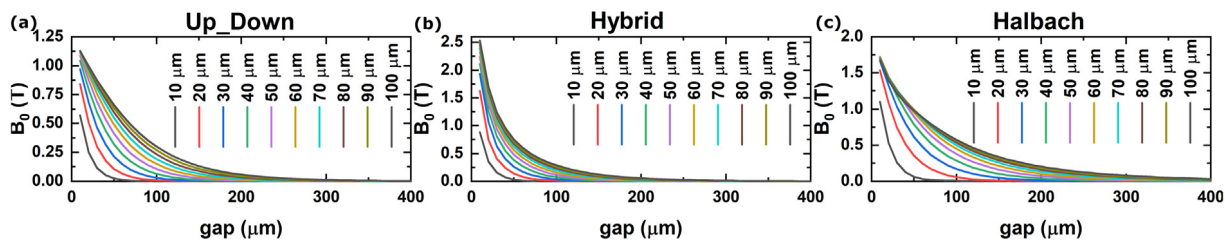


Fig. 9. (color online) Reaction of B_0 to change in gap and micro magnet thickness.

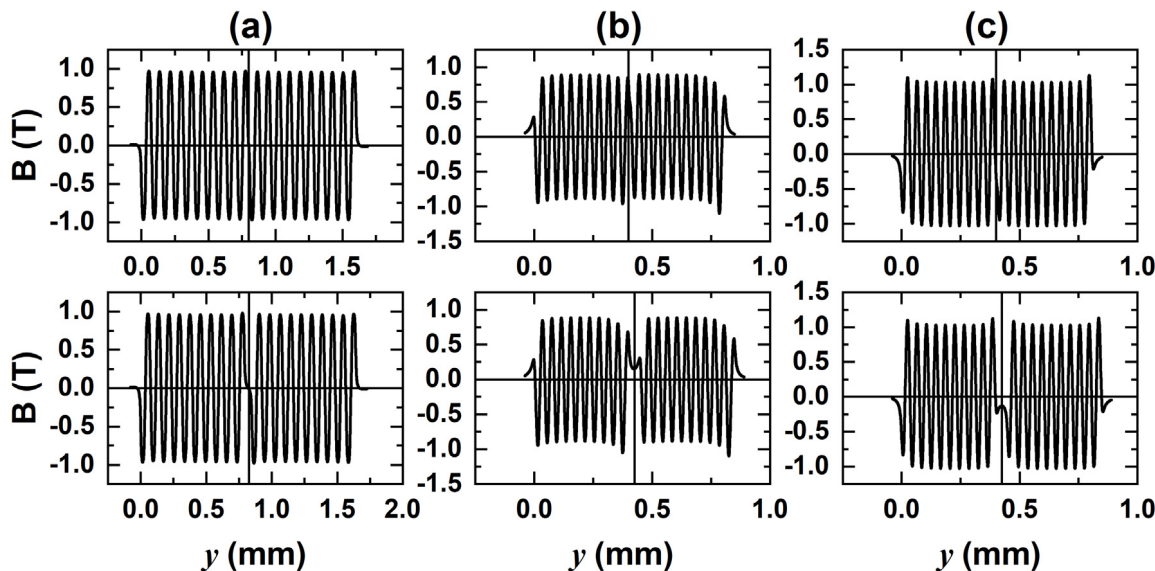


Fig. 10. Magnetic field snapshots of two consecutive undulator segments. The separation between two segments is 10 and 50 μm for the graphs on the upper and lower rows, respectively; (a) up-down, (b) hybrid, and (c) Halbach configurations. each segment is 10 periods, gap and the thickness of the magnet is 10 μm .

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